

THE OPEN UNIVERSITY OF SRI LANKA
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
DIPLOMA IN INFORMATION SYSTEMS AND TECHNOLOGY
ECZ3262 – MATHEMATICS



FINAL EXAMINATION - 2014/ 15

CLOSED BOOK

Date: 28th September 2015

Time: 0930 – 1230 hrs

READ THE FOLLOWING INSTRUCTIONS BEFORE ANSWERING THE PAPER

Instructions:

1. Answer any **five** out of eight questions.
2. Show **intermediate steps** clearly.
3. **Programmable** calculators are **not** allowed.

Q1

(a) Use Demorgan's theorem to simplify the following expressions: [06]

i) $\overline{(\overline{a+b+c})\overline{bc}+a}$ ii) $\overline{(\overline{c+a+b})(\overline{a+b})}$

iii) $\overline{(x+y+z)+(\overline{x+y+z})+(\overline{x+y+z})}$

(b) Use Truth tables to show the followings: [06]

i) $\overline{(\overline{a+b})(\overline{a+b})} = a$ ii) $(x+y)(\overline{x+y}) = xy + \overline{x+y}$

(c) Setup *K*-map table for the following truth table and then find the simplified Boolean expression for the given result **R**. [08]

A	B	C	D	R
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

Q2

- (a) If both A and B are $n \times n$ matrices with A being a non-singular, show that [02]

$$(A^{-1}BA)^2 = A^{-1}B^2A$$

- (b) Let P is a 3×3 matrix as [08]

$$P = \begin{pmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ b & c & 1 \end{pmatrix}$$

Find

- i) the matrix P^2 ;
 ii) the relation between a, b and c which gives $P^2 = I$, the unit matrix.
- (c) Consider the matrix Q as [10]

$$Q = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$$

- i) Find the inverse matrix, Q^{-1} using **Gaussian elimination** method.
 ii) Then, solve the following linear equations using above i).

$$x + y + z = 5$$

$$x + 2y + 3z = 13$$

$$x + 4y + 9z = 35$$

Q3

- (a) Answer the followings: [06]

- i) Write the product relation for $\cos(A+B)$
 ii) Prove that $\cos 3A = 4\cos^3 A - 3\cos A$ using above i)
 iii) Hence, find the value of $\cos 135^\circ$ using above ii)

- (b) Prove the followings: [08]

i) $(\cos A + \cos B)^2 + (\sin A - \sin B)^2 = 2[1 + \cos(A+B)]$

ii) $\frac{\sin \theta + \tan \theta}{\cot \theta + \operatorname{cosec} \theta} = \sin \theta \cdot \tan \theta$

- (c) Find the general solution of the following equation for $0^\circ < \theta < 360^\circ$. [06]

$$\sin^2 \theta + 2\cos \theta = -2$$

Q6

(a) Find the following limits:

[06]

$$\text{i) } \lim_{x \rightarrow 0} \frac{\sin 5x}{3x} \quad \text{ii) } \lim_{x \rightarrow 9} \frac{2x^2 - 162}{\sqrt{x} - 3} \quad \text{iii) } \lim_{x \rightarrow -2} \frac{x^3 + 8}{\frac{1}{2} + \frac{1}{x}}$$

(b) Let A and \bar{A} are the exact answer and the computed answer, respectively.Find the **absolute error** and the **relative error** when

[04]

i) $A = -0.0047$, $\bar{A} = -0.0045$

ii) $A = -0.671 \times 10^{12}$, $\bar{A} = -0.0669 \times 10^{13}$

(c) The difference table for $f(x) = e^x$ with $h = 0.2$ is shown below.

[10]

x_j	f_j	Δf_j	$\Delta^2 f_j$	$\Delta^3 f_j$	$\Delta^4 f_j$
0.0	1.0000				
		0.2214			
0.2	1.2214		0.0490		
		0.2704		0.0109	
0.4	1.4918		0.0599		0.0023
		0.3303		0.0132	
0.6	1.8221		0.0731		0.0031
		0.4034		0.0163	
0.8	2.2255		0.0894		0.0033
		0.4928		0.0196	
1.0	2.7183		0.1090		0.0047
		0.6018		0.0243	
1.2	3.3201		0.1333		
		0.7351			
1.4	4.0552				

Select $x_0 = 0.4$ and find the value $f(0.43) = e^{0.43}$ by using **Newton's forward formula**.

Q7

(a) Consider the following data set:

[10]

23	37	41	69	29	71	87	94	102
121	88	99	107	116	124	78	89	96
97	116	124	29	39	52	67	54	59

- Construct the '**Ordered stem and leaf**' plot.
- Find mode, median, and mean of the data set.

(b) A continuous random variable, X has the probability density function, $f(x)$ as

$$f(x) = \begin{cases} \frac{6}{7}x & ; 0 \leq x \leq 1 \\ \frac{6}{7}x(2-x) & ; 1 \leq x \leq 2 \end{cases}$$

Find the followings of the random variable X : [10]

- i) Expected value, $E(X)$ ii) Variance, $V(X)$

Q8

(a) Assume that the length of time, x , between charges of a pocket calculator is normally distributed with a mean of 100 hours and a variance of 225 hours. Find the probability that the calculator will last between 80 and 110 hours between charges. [08]

(b) A hospital uses large quantities of packaged doses of a particular drug. The individual dose of this drug is 100 cm^3 . The action of the drug is such that the body will harmlessly pass off excessive doses. On the other hand, insufficient doses do not produce the desired medical effect, and they interfere with patient treatment. The hospital has purchased its requirements of this drug from the same manufacturer for a number of years and knows that the population standard deviation is 2 cm^3 . The hospital inspects 50 doses of this drug at random from a very large shipment and finds the mean of these doses to be 99.75 cm^3 . If the significance level, $\alpha = 0.10$ and $Z_{0.10} = -1.28$ then answers the followings. Assume the system has a normal distribution. [12]

- i) Write the null hypotheses (H_0) and the alternative hypotheses (H_1).
- ii) Is this a two-tailed or a one-tailed test? Give the reasons.
- iii) Find the critical values.
- iv) Hospital asks, whether the dosages in this shipment are too small. What is your conclusion? Explain briefly.

End.

