

THE OPEN UNIVERSITY OF SRI LANKA
 COMMONWEALTH EXECUTIVE MASTER OF BUSINESS/PUBLIC
 ADMINISTRATION
 FINAL EXAMINATION – DECEMBER 2013
 MCP 1607 – QUANTITATIVE TECHNIQUES FOR MANAGERS
 DURATION THREE (03) HOURS



DATE: 14TH DECEMBER 2013

TIME: 9.30AM – 12.30 PM

INSTRUCTION TO CANDIDATES

- a) Answer any five (05) questions only.
- b) Each question carries equal marks.
- c) Write your index number on every page.
- d) Use of non programmable calculators are allowed.
- e) Necessary statistical tables and mathematical formulae are attached.

Q (1) a) Find the differential coefficient of the following functions with respect to "x".

(i) $2x^3 + 3x^2 + 7x + 2$ (ii) $(x^3 + 3)(x^2 + 7x + 1)$ (iii) $\frac{x}{(x+1)}$

b) Find the integral of the following functions with respect to "x".

(i) $x^2 + 3x + 2$ (ii) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (iii) $\frac{x^2+9x+14}{x+2}$

c) Solve the following definite integral.

$$\int_1^3 (3x^2 + 2x + 4) dx$$

d) The total cost of producing "x" units of a certain item is given by "Z" where;

$$Z = 2x^2 + 7x + 4$$

Find the Total Cost, Average Cost and Marginal Cost when 50 units are produced.

e) A manufacturing organization observes that their monthly profit would depend on the money spent on advertising. The relationship between profit and money spent on advertising is given by the following equation, where "P" is the profit measures in Rs."000" and "x" is cost of advertising measured in Rs. "000".

$$P = 700 + 120x - 3x^2$$

Find how much money should be spent on advertising to maximize profit.

- Q2. a) A and B are two matrices defined as follows.

$$A = \begin{pmatrix} 3 & 7 & 4 \\ 6 & 5 & 2 \\ 8 & 6 & 9 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 5 & 1 \\ 3 & 2 & 1 \\ 6 & 5 & 4 \end{pmatrix}$$

Evaluate the following.

- (i) $A + B$
 - (ii) $A - B$
 - (iii) $2 \times A$
 - (iv) $A \times B$ (Vector multiplication)
- b) Using Cramer's rule in Matrices algebra or Inverse Matrices method solve the following simultaneous equations.

$$3x + 4y + Z = 14$$

$$x + 5y + 3Z = 20$$

$$2x + y + 2Z = 10$$

- Q3. a) A dice is thrown twice. What is the probability that the sum of scores is greater than eight?
- b) Three players A, B, and C toss a coin repeatedly in the order "A", "B" and then "C" game. The first to throw a "Head" wins the game. Find the probability that "A" wins the game.
- c) In a community, 60% of the applicants for driving license attend driving school while the balance 40% learn on their own. The probability that the applicant is successful in obtaining the driving license is 0.7 for those who attend driving schools and it is 0.4 for those who learn on their own. An applicant is selected at random.
- (i) What is the probability that the applicant is successful in obtaining driving license?
 - (ii) Given that the applicant randomly selected is successful in obtaining driving license, what is the probability that he attended driving school?
 - (iii) Given that the applicant is not successful in obtaining driving license, what is the probability that he learned driving on his own?

- Q4. a) (i) Evaluate the expression ${}^n C_r p^r q^{(n-r)}$ Where $n = 5$, $r = 3$ and $p = 0.2$
- (ii) A telephone switch board has six telephone lines. The probability that any of the lines is busy is 0.7.
- (1) What is the probability that exactly two lines are busy?
- (2) What is the probability that all six lines are busy?
- b) (i) Evaluate the expression $e^{-a} \frac{a^x}{x!}$
Where $a = 2$ and $e = 2.17$
- (ii) A factory manager observes that on the average there are three machine break downs per day. What is the probability that there are two machine break downs the next day?
- c) The time taken by an officer to travel from home to office is normally distributed with mean 150 minutes and standard deviation 30 minutes.
- (i) If the officer leaves home at 6 am and office open at 9 am what is the probability that he will be late to office?
- (ii) At what time should he leave home to be 95% sure that he will not be late to office?
- Q5. a) To estimate the life span of electric bulbs, a sample of 50 bulbs were subjected to a test. It was observed that the mean and standard deviation of the life span of 50 bulbs were 1500 hours and 350 hours respectively.
- (i) Develop a 95% confidence interval estimate for life span of bulbs.
- (ii) Develop a 80% confidence interval estimate for life span of bulbs.
- b) A bakery owner maintains that the mean weight of his bread is 450 grams and it is neither more nor less. To test his statement a sample of 36 was analysed and it was observed that the mean and standard deviation of weight of the sample were 427 grams and 48 grams respectively.
- (i) State the null hypothesis and alternate hypothesis of the test.
- (ii) Carry out a statistical test at 5% level of significance.

- c) It is proposed to conduct a sample survey to estimate the mean monthly income of a house-hold in a community. A pilot survey suggests that the mean and standard deviation of monthly income of a house hold in the community could be around Rs. 7000 and Rs. 350 respectively. What should be the sample size to be 95% sure that the sampling error would not exceed 1%?

Q6. A department store gives in service training to its salesman followed by a test to consider whether they should terminate the services of any salesman who does not qualify the test. The following table gives the test score and sales made by nine salesmen along with the calculated " x^2 ", " y^2 " and " xy " terms. Variable " x " denotes test score while " y " denotes sales (Rs.000).

x	y	x^2	y^2	xy
14	31	196	961	434
19	36	361	1296	684
24	48	576	2304	1152
21	37	441	1369	777
28	50	784	2500	1400
22	45	484	2025	990
15	33	225	1089	495
20	41	400	1681	820
<u>19</u>	<u>39</u>	<u>361</u>	<u>1521</u>	<u>741</u>
<u>182</u>	<u>360</u>	<u>3828</u>	<u>14746</u>	<u>7493</u>

- Calculate the correlation coefficient between test score and sales.
- Evaluate the line of regression of the form $y = a + bx$.
- If the firm wants a minimum sales volume of Rs.30,000, what should be the test score to continue service without termination?
- What is the residual of the observation where x (TEST SCORE) is 15?
- Evaluate the sum of squares error given as "SSE".
- Calculate the coefficient of determination.
- Calculate the standard error of the "b" coefficient given by SB_1 .
- Develop a 95% confidence interval estimate for the "b" coefficient.

Q7. Write short notes on the following.

- Multicollinearity
- Residual Analysis
- Convolutions
- Exponential Smoothing

MATHEMATICS FORMULAE

i. Correlation Coefficient.

$$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\left[\sum x^2 - \frac{(\sum x)^2}{n} \right] \left[\sum y^2 - \frac{(\sum y)^2}{n} \right]}}$$

ii. Line of regression $y = a + bx$

$$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2} \quad a = \frac{\sum y}{n} - b \frac{\sum x}{n}$$

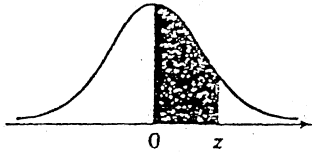
iii. SSE = sum of all (residual)² Terms.iv. Coefficient of determination = r^2

v.
$$S_{yx} = \sqrt{\frac{SSE}{(n-2)}}$$

vi.
$$SB_1 = \frac{S_{yx}}{\sqrt{SSX}} \text{ where } SSX = \sum (x_i - \bar{x})^2 = \sum x_i^2 - n\bar{x}^2$$

vii. Standard error = $\frac{S}{\sqrt{n}}$

Normal Curve Areas

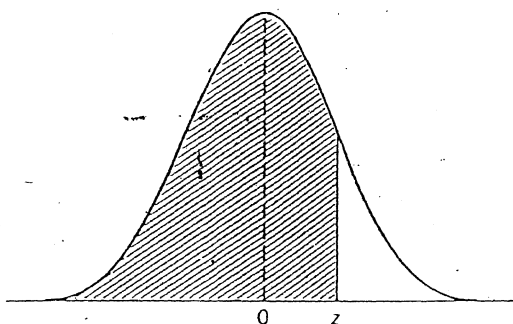


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

Table A2. Values of z , the standard normal variable, from 0.0 by steps of 0.01 to 3.9, showing the cumulative probability up to z . (Probability correct to 4 decimal places).

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
.9	1.0000									



The curve is $N(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1.96$ the shaded area is 0.9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.