THE OPEN UNIVERSITY OF SRI LANKA
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
DIPLOMA IN INFORMATION SYSTEMS AND TECHNOLOGY
ECZ3262 – MATHEMATICS
FINAL EXAMINATION - 2015/ 16 CLOSED BOOK

Date: Sunday, 27 November 2016 **Time:** 0930 – 1230 hrs

READ THE FOLLOWING INSTRUCTIONS BEFORE ANSWERING THE PAPER

Instructions:

1. Answer any **five** out of eight questions.
2. Show **intermediate steps** clearly.
3. **Programmable** calculators are **not** allowed.
4. Total marks obtaining for this examination is 100. The marks assigned for each question is in square brackets.

Q1

(a) Find which of the following propositions are tautologies, contradictions or neither. **[6]**

i) $(p \leftrightarrow q) \leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$

ii) $(p \vee q) \wedge [(\neg p) \wedge (\neg q)]$

iii) $(p \rightarrow q) \vee (q \rightarrow p)$

(b) Use DE Morgan's theorem to simplify the following expressions. **[6]**

i) $\overline{\overline{A + BC} + \overline{AB}}$

ii)) $\overline{(\overline{x} + z)(\overline{xy})}$

(c) Minimize the following using the Karnaugh maps method (first constructs the truth table and then find the simplified Boolean expression for the given expression). **[8]**

$$R = \overline{ABC} + \overline{A}B + AB\overline{C} + AC$$

Q2

(a) i) Solve $\begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix} + X = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$, where X is a matrix. **[3]**

ii) Show that $A = \begin{pmatrix} 0 & 0 \\ 1 & 3 \end{pmatrix}$ has no inverse. **[3]**

(b) Compute the following, if possible, for the given matrices. **[6]**

$$A = \begin{pmatrix} -4 & 2 & 3 \\ 0 & 5 & -1 \\ 6 & 1 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 6 & -1 & 0 \\ 2 & 2 & -4 \\ 3 & -1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 5 & -1 \\ -3 & 4 \end{pmatrix}$$

$$D = \begin{pmatrix} -7 & 1 & -4 \\ 3 & -2 & 8 \end{pmatrix} \quad E = \begin{pmatrix} 3 & -3 & 5 \\ 1 & 0 & -2 \\ 6 & 7 & -2 \end{pmatrix}$$

i) A+B

ii) C+D

iii) A^T+E^T

(c) Use Gaussian elimination to solve the system of linear equations. **[8]**

$x_1 - 2x_2 - 3x_3 = 0$

$2x_2 + x_3 = -8$

$-x_1 + x_2 + 2x_3 = 3$

Q3

(a) Find the first derivative of the followings from first principles. **[6]**

i) $\sec x$ ii) $2x + \frac{1}{x^2}$

(b) Find $\frac{d}{dx}$ as a function of x for followings. **[6]**

i) $e^{5x^2+7x-13}$ ii) $x^2 \sin^3(5x)$ iii) $\frac{1+\ln x}{x^2 - \ln x}$

(c) Show that $y = \cos^3 x \tan x$ satisfies $\cos x \frac{dy}{dx} + 3y \sin x - \cos^2 x = 0$ **[8]**

Q4

(a) Integrate following by parts. **[6]**

i) $\int x^2 e^{3x} dx$ ii) $\int x \cos 3x dx$ iii) $\int x^4 \ln x dx$

(b) Evaluate following definite integrals. **[8]**

i) $\int_0^{\frac{\pi}{3}} \frac{\cos x + \cos x \tan^2 x}{2\sec^2 x} dx$ ii) $\int_0^2 x(x^2-1)^7 dx$ iii) $\int_1^4 \frac{x^3-8}{\sqrt{x}} dx$

(b) If $\int_0^{\frac{\pi}{2}} x \sin x dx = \int_0^2 (kx^2 + 2x) dx$ find the value of **k**. **[6]**

Q5

(a) Prove the trigonometric relation **[6]**

i) cotx + tanx = secx.cosecx

ii) $\frac{1+\cos x + \cos 2x}{\sin x + \sin 2x} = \cot x$

iii) $\frac{(\tan x + \sec x - 1)}{(\tan x - \sec x + 1)} = \frac{(1+\sin x)}{\cos x}$

(b) i) Calculate the trigonometric ratios of $\sin 15^\circ$ and $\cos 15^\circ$, by using the trigonometric ratio values of 45^0 and 30^0

[4]

ii) A block of flats is 200 m away from a cellphone tower. Someone stands at **B** and measure the angle from **B** (top of the flat which is at a lower level than tower top) to the top of the tower **E** to be 30° (the angle of elevation) as shown in Figure Q5. Then they measure the angle from **B** to the bottom of the tower **C** to be 60° (the angle of depression). What is the height of the cellphone tower (Height of CE)?

[2]

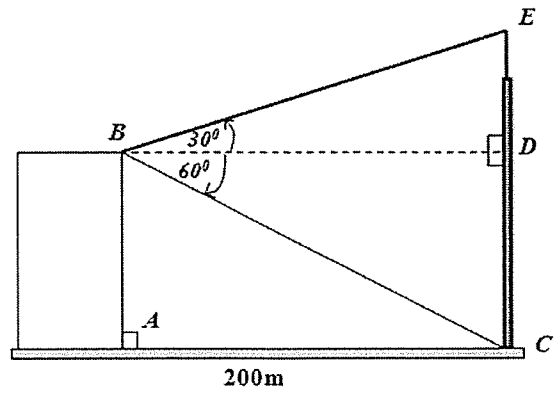


Figure Q5

(c) Sketch the graphs of the following functions for the values of x lying between 0 and 2π.

i) y=cos2x ii) $y = -3\sin\left(\frac{1}{2}x\right)$ **[8]**

Q6

(a) If $2\cos x = \left(x + \frac{1}{x}\right)$ then prove that $\cos 3x = \frac{1}{2}(x^3 + \frac{1}{x^3})$ and $\cos 6x = \frac{1}{2}\left(x^6 + \frac{1}{x^6}\right)$ **[6]**

(b) (i) Given the statistical distribution of the table. Calculate the **mode, median, mean, range, variance and standard deviation.** **[6]**

x_i	61	64	67	70	73
f_i	5	18	42	27	8

(ii) The following table shows the recorded high temperatures for each of the 50 states in US. Construct a histogram and a frequency polygon to represent data. **[3]**

Class Boundaries	Frequency
99.5 - 104.5	2
104.5 – 109.5	8
109.5 – 114.5	18
114.5 – 119.5	13
119.5 – 124.5	7
124.5 – 129.5	1
129.5 – 134.5	1

(c) ABC (PVT) LTD has a contract to assemble components for a waste water management system to be used by the Sri Lankan government. The time required to complete one part of the assembly is thought to be normally distributed, with a mean of 30 hours and a standard deviation of 4.7 hours. Find the probability that the assembly steps completed between 26 and 35 hours. **[5]**

Q7

(a) Bernad tries to read 240 pages of her book every day. One week (7 days) she only managed to read 200 pages. What was the percentage error? **[4]**

(b) The sales of the Maclak (PVT) LTD for last five years is given in the table below. Estimate the sales for the year 1979. Use the Newton-Gregory formula of backward interpolation for the calculation. **[8]**

Year	1974	1976	1978	1980	1982
Sales (in lacks)	40	43	48	52	57

(c) A continuous random variable, X has the probability density function, $f(x)$ as

$$f(x)=\begin{cases}0.5x & ;0\le x\le 2\\ 0 & ;x<0 \quad or \quad x>0\end{cases}$$

Find the followings of the random variable X : **[8]**

i) Expected value, $E(X)$ ii) Variance, $V(X)$

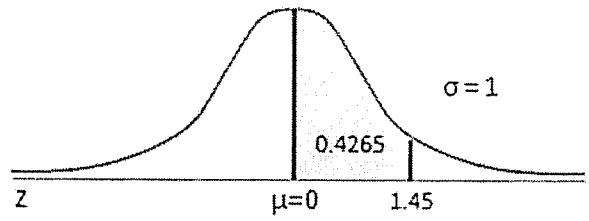
Q8

(a) There is a huge cone with the dimensions as; radius of 4 meters and height of 18 meters. You would be able to fill the cone with water at a rate of 3 cubic meters every 50 minutes. How long will it take you to fill the cone? **[8]**

(b) In a hospital union claimed nurses earned LKR 45000 with a standard deviation of 3000. A recent sample of 120 nurses found the mean income to be LKR 45500. At the $\alpha = 0.01$ level of significance; then can we conclude the mean income is not equal to LKR 45000. Explain briefly. Find the p-value. (Assume the system has a normal distribution) **[12]**

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

-END-