**THE OPEN UNIVERSITY OF SRI LANKA**
**Faculty of Engineering Technology**
**Department of Electrical & Computer Engineering**

**Diploma in Information Systems and Technology**

**Final Examination (2016/2017)**
**ECZ3262 – Mathematics**

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**Date: 26th November 2017 (Sunday)** **Time: 0930 hrs – 1230 hrs**

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***Instructions:***

- *Answer **five** questions only.*
- *Show intermediate steps clearly.*
- *Programmable calculators are not allowed.*
- *The number of questions of the paper is eight (08).*
- *The number of pages of the paper is six (06).*

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**Q1**

(a) Let $A$, $B$, $C$ and $D$ be Boolean variables. Prove the following identities.

(i) $(\overline{\bar{A}.B + B.\bar{C}}).\overline{A.\bar{B}} = \bar{A}.\bar{B} + A.B.C$

(ii) $AB + A(B + C) + B(B + C) = B + A.C$

(iii) $(A\bar{B}(C + BD) + \bar{A}\bar{B})C = \bar{B}C$ [9 marks]

(b) Let $p$, $q$ and $r$ be propositions. By constructing truth tables, show that the following propositions are equivalent.

(i) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

*(ii)* $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$ [6 marks]

(c) By using the Karnaugh maps method, minimize the following.

$\bar{P}.\bar{Q}.\bar{R} + \bar{P}.Q.\bar{R} + P.Q.\bar{R}$ [5 marks]

**Q2**

(a) If $A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$, then prove that $A^2 = 7A + 2I$, where I is the 2x2 identity matrix. [3 marks]

(b)

(i) Let $A = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$. Is the matrix A nilpotent ? Justify your answer. [3 marks]

(ii) If $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 \\ 1 & 4 \\ 5 & 2 \end{bmatrix}$ then find AB. [3 marks]

(iii) If $A = \begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 7 \\ -4 & -7 & 0 \end{bmatrix}$, prove that $A^T + A = \bar{0}$, where $\bar{0}$ represents 3x3 zero matrix. [3 marks]

(c) Using the method of Gaussian elimination, solve the following system of linear equations.

$$4x + 8y - 4z = 4$$
$$3x + 8y + 5z = -11$$
$$-2x + y + 12z = -17$$

[8 marks]

**Q3**

(a) Find the equation of the tangent to the curve of $y = 2x^3 + 3x + 7$ at the point x =2. [6 marks]

(b) Find $\frac{dy}{dx}$ for the following.

(i) $y = 5x^2e^{3x}$

(ii) $y = \frac{sin3x}{4+5cos2x}$ [8 marks]

(c) If $y_n = tan^n x$, then prove that $\frac{dy_n}{dx} = n(y_{n-1} + y_{n+1})$. [6 marks]

**Q4**

(a) Find the following indefinite integral.

$$\int \frac{1}{x^3-1}\,dx$$ [6 marks]

(b) Using integration by parts, find the following indefinite integral.

$$\int sinx\ ln(cosx)\,dx$$ [8 marks]

(c) Evaluate the following definite integral.

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} sin^3 x cosx dx$$ [6 marks]

**Q5**

(a) Sketch the graph of the following function for $x \in [0, 2\pi]$.
Write-down the amplitude and period.

$$y = 2cos\left(x - \frac{\pi}{2}\right) + 1$$ [6 marks]

(b)

(i) Solve the equation $5sinx - 2cos^2x - 1 = 0$ for $0 \leq x < 360^0$.

[4 marks]

(ii) The angles of depression of the top and the bottom of a 12 m high tree from the top of the building are 45 degree and 60 degree respectively. Calculate the height of the building.

[4 marks]

(c) Prove the following identities.

(i) $tan^4x + tan^2x = sec^4x - sec^2x$

(ii) $(1 - cos^2x)(1 + cos^2x) = 2sin^2x - sin^4x$

(iii) $\frac{1+tan^2x}{1-tan^2x} = sec2x$

[6 marks]

**Q6**

(a) Make a stem and leaf plot for a test scores given below and answer the following questions.

| 54, 65, 98, 82, 63, 71, 78, 77, 85, 95, 92, 58, 69, 70, 81, 93, 76, 82, 85, 91, 93, 99, 73 |
|---|

(i) What type of graph does a stem and leaf plot represent when turned it vertically?
(ii) What was the lowest score on the test?
(iii) What was the highest score on the test?
(iv) In which interval did most students score?

[6 marks]

(b) Find the mode, median, variance and standard deviation for the following data:
57, 64, 43, 67, 49, 59, 44, 47, 61, 59 [8 marks]

(c) The weights of bags of rice may be assumed to be normally distributed with mean 25.8kg and standard deviation 0.5kg.
(i) Determine the probability that a randomly selected bag will weigh less than 25kg.
(ii) Determine the probability that a randomly selected bag will weigh between 25.5kg and 26.5kg.

[6 marks]

**Q7**

(a)
(i) Calculate the relative error of $0.94 \pm 0.2$. [2 marks]
(ii) The density of lead is 13.6 $gcm^{-3}$, but the measured and calculated value in lab was 12.9 $gcm^{-3}$. What is the percentage error? [3 marks]

(b) Construct Newton's forward difference table for the following data.

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 4 | 9 | 16 | 25 |

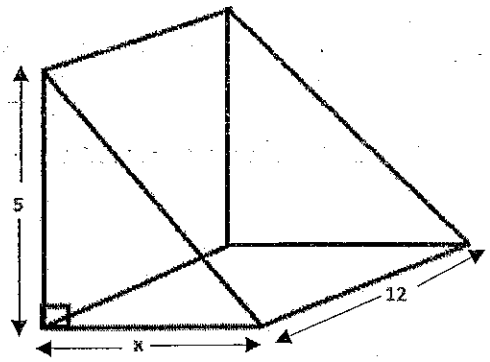
Hence calculate an approximate value for $f(1.7)$ from Newton's forward interpolation method.

[10 marks]

(c) Let $f(x) = x^3 - 2x - 5$, where $x \in R$
(i) Find $f'(x)$. [1 Marks]
(ii) Show that $f(x)$ has a real root that lies between 2 and 3. [1 Marks]
(iii) Using Newton-Raphson method find the above real root to correct to five decimal places. [3 Marks]

**Q8**

(a) The volume of the right triangular prism as shown in the diagram is 120 $cm^3$. Find the value of $x$.




[6 marks]

(b) The managing director of a company claims that the employees in his company are above average intelligence. A random sample of 30 employees of the above company IQ scores have a mean score of 112. Mean of the population IQ is 100 with a standard deviation of 15. Assuming IQ scores are normally distributed and the significant level $\alpha = 0.05$,

(i) Write the null hypotheses ($H_0$) and the alternative hypotheses ($H_1$).

[4 marks]

(ii) Is this a two-tailed or a one-tailed test? Give the reasons.

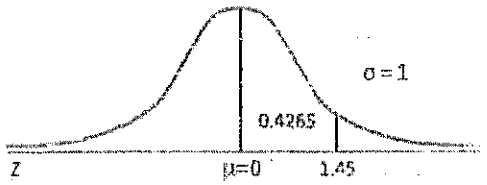
[4 marks]

(iii) Is there sufficient evidence to support the managing director's claim? Explain briefly.

[6 marks]

## Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.




| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

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