THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY– FOUNDATION (LEVEL 01)

FINAL EXAMINATION 2005

MPZ 1331/MPF 1331 – APPLIED MATHEMATICS II

DURATION – THREE (03) HOURS

DATE : 19th April, 2006 **TIME: 09.30 a.m. – 12.30 p.m.**

ANSWER (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS. YOU CANT USE MOBILE PHONES AS CALCULATORS.

SECTION – A

01. a) A system of coplanar forces has anticlock wise moments $G, 2G/3, \frac{3G}{2}$ respectively about the points 0(o,o), A (O,H), b(2H,0) IN ITS PLANE. Calculate the magnitude of the resultant of the forces and prove that the equation of its line of action is 3x-4y+12h=0.

b) Forces of magnitudes 1,5,6,11,7 and 3 act in the sides $\vec{AB}, \vec{BC}, \vec{CD}, \vec{DE}, \vec{EF}$ and $\vec{FA}$ of a regular hexagon ABCEDF of side a in the senses indicated by the order of the letters. Taking as axes OA and OL, where O is the center of hexagon and L is the mid point BC. Find the magnitude direction and line of action of the resultant force.

Prove that the line of action of the resultant force meets the axes at the points (-9a/4,0) and (0, -9 $\sqrt{3}$a/2).

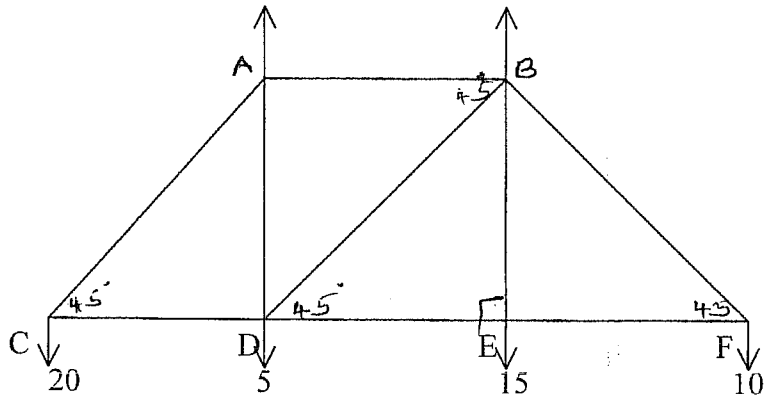
02. a) Four forces act along each side of a plane quadrilateral.
If the forces all act the same way round the quadrilateral and each is represented in magnitude by the side in which it acts, show that the resultant is a couple, whose moment is represented by twice the area of the quadrilateral.

b) A uniform bar 0.6m long and of mass 17kg., is suspended by two vertical strings. One is attached at a point 7.5cm from one end, and can just support the weight of a mass of 9kg. without breaking, the other is attached 10cm from the other end, and can just support a mass of 10kg. A weight of mass 1.7kg is now attached to the rod find the limits of the position in which it can be attached without breaking either string.

03. A uniform thin smooth rod, of weight of W and length 4a is freely hinged to a rough horizontal surface. The rod is lifted and a rough cube, of side 2a and weight W, is pushed under it until the rod is inclined at 60^0 degrees to the horizontal, show that when released the system will remain rest provided $\mu \geq \frac{3}{61}(8-\sqrt{3})$ being the coefficient of friction between the cube and the surface.

Find the horizontal and vertical components of reaction at the hinge in this position.

04. The framework ABCDEF is composed of light rods smoothly jointed, it is hung from smooth pins at A,B and carries weight as shown. Find stresses in all the rods to the nearest unit and showing whether they are tensions or thrust.



SECTION – B

05. A train runs in 10 min from rest at one station to rest at another station, distance 12Km. For the first 400m it has a constant acceleration f kmh^{-1} min^{-1} during the time period t_1 min. The next 1 Km it has a constant acceleration $\frac{f}{2}$ Kmh^{-1} min^{-1} during the time period t_2 min. It then runs at a constant maximum speed V Kmh^{-1} during time period t_3 min until the last 2 Km over which it travels with constant retardation.

Sketch the velocity time graph.
Hence show that

I. i. $t_1 = \sqrt{\frac{48}{f}}$

ii. $t_2 = \frac{2(\sqrt{108} - \sqrt{487})}{\sqrt{f}}$

iii. $t_3 = \frac{516}{\sqrt{108f}}$

II. Find the retardation time of train in terms of f.III Show that $f = \frac{8100}{108}$ $kmh^{-1}min^{-1}$.

(IV). Also show that V=90 kmh^{-1} and the retardation of the train is $33.75kmh^{-1}min^{-1}$.

06. An eroplane is to fly from a point A to a town B, 480km due north of A. If a wind of $48kmh^{-1}$ blows from the North-West find the direction in which the plane should be steered and the time T taken to reach B if the speed of the plane relative to the air is $240kmh^{-1}$.

If after the plane has flown for one-half of the time of T, the wind veers round so as to blow at $48kmh^{-1}$ from North – East, find how far from B the plane will be after time T from the start, assuming that no change in the steering of the plane is made.

07. a) A missile is projected with an initial speed 91 m/s at an angle of $\text{Sin}^{-1}\frac{12}{13}$ with the horizontal. Find its range and time of flight. Three seconds later another missile is projected from the same point and hits the ground at the same time and in the same place as the first missile, find its speed and angle of projection.

b) A light inextensible string lies on a smooth horizontal table with its ends passing over opposite edges. To these ends are attached masses M and 2m respectively, while a mass M is tied to the mid-point of string on the table. The system is released and when the strings are tight the particles move with acceleration f. Show that $f = g\left[\frac{m}{3m+M}\right]$ and find the tensions in the strings.

g - **Acceleration due to gravity – $9.8ms^{-2}$**

08. a) Three equal spheres centers A,B,C lie at rest in a straight line, on a smooth horizontal table. The coefficient of restitution between any two spheres is e. A is projected with speed U to strike B directly. Find the velocities of A and B after this impact. After that B directly strikes C. Deduce the velocities of B and C after this impact. And show that A strikes B again whatever the value of e.

b) A train of total mass 160 tonne is ascending a hill inclined at $\sin^{-1}(1/280)$ with the engine working at half its maximum rate of 420kW. The train moves steadily at 63km/h. Find the resistance to motion. If the engine is now mode to work at maximum rate, find the immediate acceleration up the hill. (Take $g=9.8ms^{-2}$).

SECTION – C

09. a) An alloy is composed of zinc and copper whose specific gravities are respectively 7 and 8.5; if the alloy is volume $0.0062m^3$, and its specific gravity is 8. What volumes of zinc and copper does it contain?

b) A cylinder of specific gravity 0.95 and height 30cm floats with it axis vertical in a vessel containing oil and water and is totally immersed. What length of the cylinder is in the oil if the specific gravity of the oil is 0.84?

10. Assuming that the depth of the center of pressure of a vertical circular area of radius a immersed with its center at a depth h is $\frac{a^2}{4h}$ below from the its center.

A light flap is used to close a circular hole of radius r, in the side of a tank. It is kept in place by bolts at the highest and the lowest points of the hole.

The tank is filled with water of density ρ to a height H above abore the highest point of the hole.

Find the trust on the circular flab. Show that forces on the lower and upper bolts are $\frac{\pi r^2}{8}(4H+5r)\rho g$ and $\frac{\pi r^2}{8}(4H+3r)\rho g$.