THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – FOUNDATION (LEVEL 01)

FINAL EXAMINATION - 2005

MPZ 1330/MPF 1330 – PURE MATHEMATICS II

DURATION – THREE (03) HOURS

DATE : 16th March 2006 **TIME: 09.30 – 12.30 p.m.**

YOU CANT USE MOBILE PHONES AS A CALCULATORS.
ANSWER (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS.

SECTION – A

01. a) Verify that the identity $(x^2-2y^2)^2 - 121x^2y^2 \equiv x^4-125x^2y^2+4y^4$
Find the factors of $x^4-125x^2y^2+4y^4$
Hence solve the following simultaneous equations.

$$a^4+4b^4-125a^2b^2 = -4160$$
$$a^2-2b^2+11ab = 52$$

(Given that a and b are positive integers)

b) Solve the equation $\sqrt{x^2+4}+\sqrt{2x^2+25}=\sqrt{12x^2-23}$

c) Solve the simultaneous equations

$$5x+4y-3z = 20$$
$$x+y-z = 4$$
$$2x+y+2z = 10$$

02. a) Show that $\log_a b . \log_b a = 1$
By using the above result and $\log_x y^n = n\log_x y$

Show that $\log_a b^2$ x $\log a^3 = 6$

(b) By using the identify $\log_a b.\log_b c.\log_c a \equiv 1$, find the expression for $\log_b c$ in the terms of $\log_a b$ and $\log_a c$. Hence solve the simultaneous equations $\log_2 x+\log_2 y=3$, $\log_y x=2$.

c) Solve for x, correct to two significant figures, the equations $4^x - 2^{x+1}-3=0$

03. a) Let.

i. $f(x) = -x^2+2x+3$

ii. $g(x) = x^2+x+1$

iii. $h(x) = x^2+4x+4$

iv. $k(x) = x^2-4x-5$

Express the above four functions, in the form of a $a[x+\lambda]^2 \pm \mu^2$ where a,λ and μ are the constants to be determine.

b) Hence sketch the graphs of the functions y=f(x), y=g(x), y=h(x) and y=k(x)
Indicate clearly in the graphs, the greates/least values of the functions, symmetrical axes of the graphs and the values of x such that f(x)=0, g(x)=0 h(x)=0 and k(x)=0.

SECTION – B

04. a) Prove the following identities;

i. $(1-\cos A)(1+\sec A) \equiv \sin A \tan A$

ii. $(\operatorname{Cosec}A-\operatorname{Sin}A)(\operatorname{Sec}A-\operatorname{Cos}A) \equiv \operatorname{Cos}A \operatorname{Sin}A$

iii. $(\sec^2\theta+\tan^2\theta)(\operatorname{cosec}^2\theta+\cot^2\theta) \equiv 1+2\sec^2\theta\operatorname{cosec}^2\theta$

iv. $\dfrac{\tan^2 A + \operatorname{Cos}^2 A}{\operatorname{Sin}A + \operatorname{Sec}A} \equiv \operatorname{Sec}A - \operatorname{Sin}A.$

b) PQR is a triangle, X is the mid point of the line QR, XA and XB are the perpendiculars to PQ and PR respectively. If PQ=PR show that AX=XB.

05. a) Find the general solutions of the following equations.

i. $Cot2\theta = Tan\theta$ ii. $Tan3\theta = \sqrt{3}$

iii. $2Cos^2x - \sqrt{3}\,sinx + 1 = 0$ iv. $Sin5x = Cos2x$

b) As x increase from $(o, 2\pi)$ rad sketch the graph of $y=f(x)=Sinx-Cosx-1$. Indicate clearly in the graph the maximum and minimum values of f(x).

i. Find he values of x, such that $f(x)=0$.

06. a) Find the values of $Tan\theta$, $Sec\theta$ and $Sin\theta$. Such that

i. $Sec\theta - Tan\theta = 5$ ii. $Sec\theta + Tan\theta = 2$

b) Given that A is the acute angle such that CosA=3/5 and B is not an acute angle.
Such that TanB=5/12.

Find without using calculators or tables.

i. TanA, SinA, Sin B and CosB

ii. Cos (A+B) and Sin (A+B)

Find the range of the angle (A+B)

iii. Find the value Tan (A+B)

SECTION – C

07. a) Evaluate the limits.

i. $\lim_{t \to \pi} \frac{(1+Cost)}{(t-\pi)^2}$ ii. $\lim_{x \to 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{x}$

b) Find the differential coefficient (derivatives) of the following with respect to x.

i. $y = e^{-bx}Sin\,ax - e^{-bx}Cosax$ ii. $y = Tan^{-1}\sqrt{\frac{b-x}{x-a}}$ $b>x>a$

c) y=Cos(Sinx) prove that

$$\frac{d^2y}{dx^2} + \tan x\frac{dy}{dx} + yCos^2x = 0$$

Hence deduce that if $y' = Cos(Cosx) then.$

$$\frac{d^2y'}{dx^2} - Cotx\frac{dy'}{dx} + y'^2Sin^2x = 0$$

08. a) Given that $y = \frac{2x}{1+x^2}$

Find the values of x for which $\frac{dy}{dx} = 0$

Hence determine the nature of those stationary values of y.
Find the behaviour of y,
When $|x|$ tends to very large values.

Sketch the curve $y = \frac{2x}{1+x^2}$

b) A cylindrical one end open vessel has to be constructed so as to hold exactly $0.5m^3$ capacity. If the amount of material area is to be kept at the minimum. Find the measurements of the vessel.

09. a) Integrate the following with respect to x.

i. $\int \frac{dx}{1 - Cos2x}$ ii. $\int (Co\sec x + Cotx)^2 dx$

iii. $\int_0^{\frac{\pi}{2}} \frac{dx}{3 + 5Sinx}$ iv. $\int e^{3x} Sin3xdx$

b) Using the substitution $x=4\ Sin^2\theta$ or other wise
Show that $\int_0^2 \sqrt{x(4-x)}dx = \pi$

c) Using the substitution z=1-x or otherwise, evaluate

$$\int_0^1 x^2(1-x)^{\frac{1}{2}} dx$$

10. a) Find the partial fractions of $\frac{2x}{(1-x)(1+x^2)}$

Hence show that $\int_0^2 \frac{2x}{(1-x)(1+x^2)} dx = \frac{1}{2}\ln 5 - Tan^{-1}2$

b) Find $\int_0^2 \frac{(x+1)dx}{\sqrt{x^2+2x+8}}$ By substituting $U=x^2+2x+8$

c) Calcualte the area bounded by the lines ox=0, x=1, y=1 and the part of the graph of $y = \frac{x^2}{x^2+1}$ between x=0 and x=1.