THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – FOUNDATION (LEVEL 01)

FINAL EXAMINATION - 2005

MPF 1301 – PURE MATHEMATICS II

DURATION – THREE (03) HOURS

DATE : 16th March 2006 **TIME: 09.30 – 12.30 p.m.**

. YOU CANT USE MOBILE PHONES AS A CALCULATORS.
ANSWER (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS.

SECTION – A

01. a) If Ur is the r^{th} term of the series

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Find the expression for Ur.

Given that $f(r)=\frac{1}{r}$ show that

$Ur=f(r)-f(r+1)$,

Hence find $\sum_{r=1}^{n} Ur$

Deduce that $\sum_{r=1}^{\infty} Ur = 1$

b) Solve the equations $x^2-xy = 6$
$y^2-xy= 10$

02. Prove the following inequalities given that p,q,r are real, $p \neq q \neq r$ and a,b,c, are positive and $a \neq b \neq c$.

a) $p^2+q^2>2pq$

b) $\frac{a+b}{2} > \sqrt{ab}$

c) $p^2+q^2+r^2>pq+qr+rp$

d) $P^3+q^3+r^3>3\,pqr$

e) $\frac{a+b+c}{3} > \sqrt[3]{abc}$

f) $\frac{1}{3}(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}) > \sqrt[3]{\frac{1}{abc}}$

(g) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) > 9$

03. a) Let.

i. $f(x) = -x^2+2x+3$

ii. $g(x) = x^2+x+1$

iii. $h(x) = x^2+4x+4$

iv. $k(x) = x^2-4x-5$

Express the above four functions, in the form of a $a[x+\lambda]^2 \pm \mu^2$ where a, λ and μ are the constants to be determine.

b) Hence sketch the graphs of the functions y=f(x), y=g(x), y=h(x) and y=k(x) Indicate clearly in the graphs, the greates/least values of the functions, symmetrical axes of the graphs and the values of x such that f(x)=0, g(x)=0 h(x)=0 and k(x)=0.

SECTION – B

04. a) Prove the following identities;

i. $(1-\text{Cos}A)(1-\text{Sec}A) \equiv \text{Sin}A \tan A$

ii. $(\text{Cosec}A-\text{Sin}A)(\text{Sec}A-\text{Cos}A) \equiv \text{Cos}A\ \text{Sin}A$

iii. $(\text{Sec}^2\theta+\text{Tan}^2\theta)(\text{Cosec}^2\theta+\text{Cot}^2\theta) \equiv 1+2\sec^2\theta\,\text{cosec}^2\theta$

iv. $\dfrac{\tan^2 A + Cos^2 A}{SinA + SecA} \equiv SecA - SinA.$

b) PQR is a triangle, X is the mid point of the line QR, XA and XB are the perpendiculars to PQ and PR respectively. If PQ=PR show that AX=XB.

05. a) Find the general solutions of the following equations.

i. $\text{Cot}2\theta = \text{Tan}\theta$

ii. $\text{Tan}3\theta = \sqrt{3}$

iii. $2\text{Cos}^2x - \sqrt{3}\sin x + 1 = 0$

iv. $\text{Sin}5x = \text{Cos}2x$

b) As x increase from $(o,2\pi)$ rad sketch the graph of $y=f(x)=\text{Sinx}-\text{Cosx}-1$. Indicate clearly in the graph the maximum and minimum values of f(x).

Find he values of x, such that $f(x)=0$.

06. a) Using the usual notation, show that for an acute triangle ABC.

i. a=bcosC+c Cos B

ii. $\frac{a}{SinA}=\frac{b}{SinB}=\frac{c}{Sinc}$

By using part (ii)

Deduce that $\frac{a^2-b^2}{c^2}=\frac{Sin(A-B)}{Sin(A+B)}$

b) One side of a rhombus lies along the line 5x+7y=1 and one of the vertices is (3,- one diagonal of the rhombus is the line 3y=x+1 find the coordinates of the other vertices and the equations of the three remaining sides.

SECTION - C

07. a) Evaluate the limits.

i. $\underset{t\to\pi}{Lt}\frac{(1+Cost)}{(t-\pi)^2}$

ii. $\underset{x\to 0}{Lt}\frac{\sqrt{1-x}-\sqrt{1+x}}{x}$

b) Find the differential coefficient (derivatives) of the following with respect to x.

i. $y=e^{-bx}\text{Sin }ax-e^{-bx}\text{Cos}ax$

ii. $y=\text{Tan}^{-1}\sqrt{\frac{b-x}{x-a}}$ b > x > a

c) y=Cos(Sinx) prove that

$$\frac{d^2y}{dx^2}+\tan x\frac{dy}{dx}+yCos^2x=0$$

Hence deduce that if $y'=Cos(COsx) then.$

$$\frac{d^2y'}{dx^2}-Cotx\frac{dy'}{dx}+y'^2Sin^2x=0$$

08. a) Given that $y = \frac{2x}{1+x^2}$

Find the values of x for which $\frac{dy}{dx} = 0$

Hence determine the nature of those stationary values of y.
Find the behaviour of y,
When $|x|$ tends to very large values.

Sketch the curve $y = \frac{2x}{1+x^2}$

b) A cylindrical one end open vessel has to be constructed so as to hold exactly $0.5m^3$ capacity. If the amount of material area is to be kept at the minimum. Find the measurements of the vessel.

09. a) Integrate the following with respect to x.

i. $\int \frac{dx}{1 - Cos2x}$ ii. $\int (Co\sec x + Cotx)^2 dx$

iii. $\int_0^{\frac{\pi}{2}} \frac{dx}{3 + 5Sinx}$ iv. $\int e^{3x} Sin3xdx$

b) Using the substitution $x=4\ Sin^2\theta$ or other wise
Show that $\int_0^2 \sqrt{x(4-x)}dx = \pi$

c) Using the substitution z=1-x or otherwise, evaluate

$\int_0^1 x^2(1-x)^{\frac{1}{2}} dx$

10. a) Find the partial fractions of $\frac{2x}{(1-x)(1+x^2)}$

Hence show that $\int_0^2 \frac{2x}{(1-x)(1+x^2)} dx = \frac{1}{2}\ln 5 - Tan^{-1}2$

b) Find $\int_0^2 \frac{(x+1)dx}{\sqrt{x^2+2x+8}}$ By substituting $U=x^2+2x+8$

c) Calculate the area bounded by the lines ox=0, x=1, y=1 and the part of the g of $y = \frac{x^2}{x^2+1}$ between x=0 and x=1.