

THE OPEN UNIVERSITY OF SRI LANKA
 DIPLOMA IN TECHNOLOGY- FOUNDATION LEVEL 02
 FINAL EXAMINATION - 2006
 MPZ 2230 - MATHEMATICS - PAPER II
 DURATION - THREE (03) HOURS



DATE : 19th March 2007

TIME: 9.30 a.m. -

ANSWER SIX (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS. YOU CANT USE MOBILE PHONES AS CALCULATORS.

SECTION - A

01. ABC is an isocoles triangle such that $AB=AC$. The equations of the line AB and BC are $2x-y-1=0$ and $x-2y+1=0$ respectively. Prove that the line AC is parallel to the line $2x+11y=0$. If $A \equiv (2,3)$. Find the coordinate of the point C. Find the area of the triangle.
02. a) The circle $S \equiv x^2+y^2+2gx+2fy+c=0$ and the straight line $l \equiv px+qy+r=0$ intersect each other at A and B. Interpret the equation $S+\lambda l=0$. Where λ is a parameter.
- Find the equation of the circle S^1 which has AB as the diameter when $S \equiv x^2 + y^2 - 6x + 2y - 17 = 0$ and $l \equiv x-y+2 = 0$, Show that the circle S^1 and the circle $x^2 + y^2 - 8x + 2y + 13 = 0$ touch externally. Show that the circle s^1 and the circle $x^2+y^2-8x+2y+13=0$ touch externally.
- b) The circle S passes through (2,0) and cuts the circle S^1 $x^2 + y^2 = 1$ at diametrically opposite points on S^1 . Find the equation of S if it cuts the circle $x^2 + y^2 - 4y - 5 = 0$ at right angle.
03. Find the equation of the chord joining the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ on the parabola $y^2 = 4ax$.
- PQ is a chord of this parabola passing through the focus S (a,0). PM and QN drawn perpendicular to line $x+a=0$ meeting it at M and N respectively. Prove that,
- (a) PN and QM passes thought the origin.
 (b) MN subtends a right angle at the focus.
 (c) The tangents to the parabola at P and Q form with SM and SN a rectangle one vertex of which lies on MN (Hint: find the coordinal of the intersecting point of tangent at P and Q).

04. **Please answer only one part from a & b**

- a) Prove that the equation of the normal at the point P (x_1, y_1) on the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2(x-x_1)}{x_1} = \frac{b^2(y-y_1)}{y_1}$. The normal at P cuts the X axis at Q and the Y axis at R. Show that PQ:PR is constant for all positions of P.

Show that as P describes the ellipse the vertex S of the rectangle OQSR, being the origin also describes an ellipse.

- b) Find the equation of the chord joining the points $(ct, c/t)$ and $(cu, c/u)$ on the rectangle hyperbola $xy = c^2$.

A variable chord of the hyperbola passes through the point $A(c, 2c)$. Show that the equation of the locus of the mid point of the chord is $2xy = c(2x+y)$ and that the locus passes through A.

The normal at A to the locus cuts the hyperbola $xy = c^2$ at B and C. Prove that $BC^2 = \frac{85c^2}{4}$.

05. a) Let $f(x) \equiv 3\cos^2 x + 2\sqrt{3} \sin x \cos x + \sin^2 x$, $x \in \mathbb{R}$. Express in the form $A + B \cos(2x - \alpha)$. Determine the values of A, B α constants.

Hence find the maximum and minimum values of $f(x)$ and corresponding values of x , $-2\pi \leq x \leq 2\pi$.

- b) i. Solve the following equations;

$$\cos 3x - \cos 5x = 2 \sin x$$

ii. Show that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$

- c) By using general notation of ABC triangle show that $a^2 + b^2 + c^2 = \frac{abc}{R} (\cot A + \cot B + \cot C)$.

SECTION - B

06. State de Moivre's Theorem,
If ω be a complex cube root of unity.
Find that ω in the form of $\cos\theta + j\sin\theta$.

Deduce that $\omega + \omega^2 + 1 = 0$

Show that

- i. $(1 - \omega + \omega^2)(1 + \omega - \omega^2) = 4$
 - ii. $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8) = 9$
 - iii. $\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} = \omega$
 - iv. $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = 0$
07. a) Z is a any complex number such that $|z| = 1$. By using argond diagram or otherwise prove that $1 \leq |z - 2| \leq 3$ and $\text{Arg } |z - 2| \geq \frac{5\pi}{6}$,
- $$\text{Arg } |z - 2| \leq -\frac{5\pi}{6}.$$
- b) The position vectors of A, B and C are respectively $\underline{a} = 12\underline{i} + 8\underline{j} + \underline{k}$,
- $$\underline{b} = 3\underline{i} + 2\underline{j} - 5\underline{k} \text{ and } \underline{c} = -3\underline{i} - 2\underline{j} + p\underline{k}$$
- i. Find the value of p which
 - (α) \underline{c} and \overrightarrow{AB} are parallel
 - (β) A, B and C are collinear.
 - ii. Show that if p is a variable, then the locus of C is a straight line, and find the two unit direction vectors along this line.

08. Define the vector product $\underline{a} \times \underline{b}$ and the scalar product $\underline{a} \cdot \underline{b}$ of two non zero vectors \underline{a} , and \underline{b} . The position vector of the points P, Q, R with respect to O origin are $\underline{p} = 2\underline{i} - 3\underline{j} + \underline{k}$, $\underline{q} = \underline{j} - \underline{i} + 2\underline{k}$ $\underline{r} = -\underline{i} + 2\underline{j} + 3\underline{k}$ respectively.

Find the position vector of the point S, such that PQRS is a parallelogram.

Also find i) $\underline{PQ} \times \underline{PS}$, $\underline{PQ} \cdot \underline{PS}$

ii) The area of the PQRS parallelogram.

iii) Find the unit vector of the normal to the PQRS plane.

SECTION - C

09. Event A and C are independent, probabilities relating to event A, B and C are as follows. $P(A) = 1/5$ $P(B) = 1/6$ $P(A \cap C) = \frac{1}{20}$ $P(B \cap C) = \frac{1}{24}$. Evaluate $P(C)$, $P(B/C)$ and $P(C/B)$ Show that events B and C are independent.

A bag initially contains 1 red ball and 2 blue balls. A trial consists of selecting a ball at random noting its colour, and replacing it together with an additional ball of same colour.

Given that three trials are made, draw a tree diagram illustrating the various probabilities. Hence or otherwise find the probability that,

- a) at least one blue ball is drawn
b) exactly one blue ball is drawn.

Given that all three balls drawn are of the same colour, find that they are all red.

10. a) The following table shows, the tensile stress of 100 steel bars. Calculate the median, mean and standard deviation.

Stress / 100N/m^2	No of bars
0 - 10	3
10 - 20	16
20 - 30	26
30 - 40	31
40 - 50	16
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- b) The mean of 48 readings of variable was 7.50 and the standard deviation was 0.30 two additional readings 6.8 and 8.2. If these are included the original, find the (a) Mean (b) standard deviation of the whole set of 50 readings.

11. a) A light inextensible string of length $3a$ has one end fixed at a point B which is vertically below A and at a distance $2a$ from it. A small ring R of mass m is threaded on the string. If R is fixed to the mid point of the string and moves in a horizontal circle with speed $\sqrt{5ga}$. Find the tensions in the parts AR and BR of the string.

- b) A particle of mass 2 kg is attached to one end of a light elastic string of natural length 1 m whose modulus of elasticity is 49 N . The other end of the string is pastened to a fixed point O. The particle is held at O and is then released from that position. Find the depth below O of the level where the particle first comes to instanceus rest. Find also the period of oscillation of the subsequent motion.

12. A uniform rod AB of mass $3m$ and length $2a$ has a particle of mass $2m$ attached to end B. The rod can rotate in a vertical plane about a smooth axis through A. If the body is slightly displaced from the position in which B is vertically above A. What is the moment of inertia of the body about the axis through A. BY applying energy conservation principle or otherwise prove that $\dot{\theta}^2 = \frac{7g}{12a}(1 - \text{Cos}\theta)$.

Where $\dot{\theta}$ is the angular velocity of the system at θ angular displacement.

Also find the magnitude of the reaction at the axis, when the rod has rotated through $\frac{\pi}{3}$ rod.

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ශ්‍රී ලංකා විවෘත විශ්වවිද්‍යාලය.

ඉංජිනේරු තාක්ෂණ ඩිප්ලෝමා පාඨමාලාව - මට්ටම 02

අවසාන පරීක්ෂණය - 2006

ගණිතය II - MPZ 2230

කාලය - පැය 03 යි.



දිනය - 2007.03.19

වේලාව - පැය 0930 සිට 1230 දක්වා

A,B හා C කොටසකින්ම අඩුම වශයෙන් එක් ප්‍රශ්නයක්වත් තෝරා ගනිමින් ප්‍රශ්න 6ක් පිළිතුරු සපයන්න. Non Programmable ගණක යන්ත්‍ර භාවිතා කළ හැක. ගණක යන්ත්‍ර සඳහා ජංගම දුරකථන භාවිතා කිරීමට ඉඩ දෙනු නොලැබේ.

A - කොටස

01. ABC සමද්විපාදී ත්‍රිකෝණයේ $AB = AC$ වේ. AB හා AC පාදවල සමීකරණ පිළිවෙලින් $2x - y - 1 = 0$ හා $x - 2y + 1 = 0$ වේ. AC රේඛාව $2x + 11y - 10 = 0$ සමාන්තර වේ. $A = (2,3)$ නම් C ලක්ෂ්‍යයේ ඛණ්ඩාංක සොයන්න. ත්‍රිකෝණයේ වර්ගඵලය සොයන්න.

02. (a) $S = x^2 + y^2 + 2gx + 2fy + c = 0$ වෘත්තය සහ $l = px + qy + r = 0$ රේඛාව A හා B ලක්ෂ්‍ය දෙකකදී ඡේදනය කරයි. λ පරාමිතියක් විට $s + \lambda l = 0$ සමීකරණය විචරණය කරන්න. $S = x^2 + y^2 - 6x + 2y - 17 = 0$ සහ $l = x - y + 2 = 0$ විට, AB විෂ්කම්භයක් වනසේ වූ S' වෘත්තයේ සමීකරණය සොයන්න.

S වෘත්තයත් $x^2 + y^2 - 8x + 2y + 13 = 0$ වෘත්තයත් බාහිරව ස්පර්ශ කරන බව පෙන්වන්න.

(b) S වෘත්තයක් $(2,0)$ ලක්ෂ්‍ය හරහා යන අතර, $x^2 + y^2 = 1$ වෘත්තයේ විෂ්කම්භයක සම්මුඛ ලක්ෂ්‍යවලදී ඡේදනය කරයි.

S වෘත්තය $x^2 + y^2 - 4y - 5 = 0$ වෘත්තය සෘජු කෝණීව ඡේදනය කරන්නේ නම් S වෘත්තයේ සමීකරණය සොයන්න.

03.

$(at_1^2, 2at_1)$ හා $(at_2^2, 2at_2)$ ලක්ෂ්‍ය යාකරන $y^2 = 4ax$ පරාවලයේ ජ්‍යායේ සමීකරණය සොයන්න.

PQ යනු මේ පරාවලයේ නාභිය S(a,0) ලක්ෂ්‍යය හරහා යන ජ්‍යායකි. $x + a = 0$ රේඛාව M හා N හිදී හමුවන සේ $x + a = 0$ රේඛාවට ලම්බ වන පරිදි PM හා QN ඇඳ තිබේ.

- (a) PN හා QM රේඛාව මූල ලක්ෂ්‍යය හරහා යන බවත්;
- (b) MN, රේඛාව නාභියේ දී සෘජුකෝණයක් ආපාතනය කරන බවත්
- (c) P හා Q හිදී පරාවලයේ ස්පර්ශක සහ SM, SN රේඛාවන් වලින් සෑදෙන සෘජු කෝණාස්‍රයේ එක් ශීර්ෂයක් MN මත පිහිටන බවත් (ඉඟිය P හා Q හිදී ස්පර්ශකවල ජේදන ලක්ෂ්‍යයේ බිඳ්විංක සොයන්න.) පෙන්වන්න.

04. කරුණාකර (a) හා (b) කොටස්වලින් එකකට පමණක් පිළිතුරු සපයන්න.

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ඉලිප්සයේ p(x,y) ලක්ෂ්‍යයේ අභිලම්භයේ සමීකරණය

$$\frac{a^2}{x_1}(x - x_1) = \frac{b^2}{y_1}(y - y_1) \text{ බව සාධනය කරන්න.}$$

P හි අභිලම්භය x අක්ෂය Q හිදීත්, y අක්ෂය R හිදීත් හමු වේ. PQ : PR අනුපාතය P හි සියළු පිහිටීම් සඳහා නියතයක් බව සාධනය කරන්න. P ඉලිප්සය මත ගමන් කරන විට OQSR සෘජු කෝණාස්‍රයක් වනසේ වූ S ශීර්ෂයද ඉලිප්සයක් මත පිහිටන බව සාධනය කරන්න. O මූල ලක්ෂ්‍යයයි.

(b) $xy = c^2$ සෘජු කෝණාස්‍ර බහුවලය මත (ct, c/t), (cu, c/u) ලක්ෂ්‍ය යාකරන ජ්‍යායේ සමීකරණය සොයන්න.

බහුවලයේ විචලන ජ්‍යායක් A(c, 2c) ලක්ෂ්‍ය හරහා ගමන් කරයි. ජ්‍යායේ මධ්‍ය ලක්ෂ්‍යයේ පථය $2xy = c(2x + y)$ බව පෙන්වා එම පථය A හරහා ගමන් කරන බවද සාධනය කරන්න.

A ලක්ෂ්‍යයේදී පථයට ඇඳී අභිලම්භය $xy = c^2$ බහුවලය B හා C ලක්ෂ්‍යවලදී හමුවේ. $BC^2 = \frac{85c^2}{4}$ බව පෙන්වන්න.

05. (a) $f(x) \equiv 3 \cos^2 x + 2\sqrt{3} \sin x \cos x + \sin^2 x$; $x \in \mathbb{R}$; $f(x) \equiv A + B \cos(2x - \alpha)$
 ආකාරයට ප්‍රකාශ කරන්න. මෙහි A, B, α නියතවල අගයන් නිර්ණය කළ යුතුය. ඒනගින් $f(x)$ ශ්‍රිතයේ වැඩිතම හා අඩුතම අගයන් හා $-2\pi \leq x \leq 2\pi$ පරාසයේ එම අගයන්ට අනුරූප x හි අගයන් ද ලබා ගන්න.

(b) (i) පහත සමීකරණ විසඳන්න.
 $\cos 3x - \cos 5x = 2 \sin x$

(ii) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{3}{5}\right)$ බව සාධනය කරන්න.

(c) ABC ත්‍රිකෝණයේ, සාමාන්‍ය අංකනයට අනුව

$$a^2 + b^2 + c^2 = \frac{abc}{R} [\cot A + \cot B + \cot C] \quad \text{බව පෙන්වන්න.}$$

B - කොටස

06. ද මූලාවර් ප්‍රමේයය ප්‍රකාශ කරන්න.

ω යනු 1 හි සංකීර්ණ සහ මූලයකි. $\omega, \cos \theta + j \sin \theta$ ආකාරයට ප්‍රකාශ කරන්න.

$\omega^2 + \omega + 1 = 0$ බව අපෝහනය කරන්න.

(i) $(1 - \omega + \omega^2)(1 + \omega - \omega^2) = 4$

(ii) $(1 - \omega)(1 - \omega^2)(1 - \omega^4)(1 - \omega^8) = 9$

(iii) $\frac{a + b\omega + c\omega^2}{b + c\omega + a\omega^2} = \omega$

(iv) $(1 + \omega - \omega^2)^3 - (1 - \omega + \omega^2)^3 = 0$

බව අපෝහනය කරන්න.

07. (a) $|Z| = 1$ වනසේ Z ඕනෑම සංකීර්ණ සංඛ්‍යාවකි. ආගන්ථි සටහන භාවිතයෙන් හෝ අන් අයුරකින්

$1 \leq |Z - 2| \leq 3$ හා

විස්තා $(Z - 2) \geq \frac{5\pi}{6}$ හා විස්තා $(Z - z) \leq -\frac{5\pi}{6}$ බව පෙන්වන්න.

(b) A, B හා C ලක්ෂ්‍යවල පිහිටුම් දෛශික

$$\underline{a} = 12\underline{i} + 8\underline{j} + \underline{k}, \quad \underline{b} = 3\underline{i} + 2\underline{j} - 5\underline{k}$$

$$\text{හා } \underline{c} = -3\underline{i} - 2\underline{j} + p\underline{k}$$

(i) (α) \underline{c} හා AB සමාන්තර වන පරිදි

(β) A, B හා C එක රේඛීය වන පරිදි p හි අගය සොයන්න.

(ii) p විචලනය වන විට C ලක්ෂ්‍යයේ පථය සරල රේඛාවක් බව පෙන්වා, රේඛාව ඔස්සේ වන ඒකක දිශා දෛශික දෙක සොයන්න.

08. \underline{a} හා \underline{b} නිෂ්ශුන්‍ය දෛශික දෙකක $\underline{a} \times \underline{b}$ දෛශික ගුණිතය හා $\underline{a} \cdot \underline{b}$ අදිශ ගුණිතය අර්ථ දැක්වන්න.

O ලක්ෂ්‍යයක් අනුබද්ධව P, Q, R ලක්ෂ්‍යවල පිහිටුම් දෛශික පිළිවෙලින්

$$\underline{p} = 2\underline{i} - 3\underline{j} + \underline{k}$$

$$\underline{q} = \underline{i} - \underline{j} + 2\underline{k}$$

$$\underline{r} = -\underline{i} + 2\underline{j} + 3\underline{k} \text{ වේ.}$$

PQRS සමාන්තරාස්‍රයක් වන පරිදි S ලක්ෂ්‍යයේ පිහිටුම් දෛශිකය සොයන්න. තවදුරටත්,

(i) $\vec{PQ} \times \vec{PS}, \vec{PQ}, \vec{PS}$

(ii) PQRS සමාන්තරාස්‍රයේ වර්ගඵලය

(iii) PQRS තලයට ඇති අභිලම්භ ඒකක දෛශික සොයන්න.

C - කොටස

09. A හා C ස්වායත්ත සිද්ධි දෙකක් වන අතර, A, B, C සිද්ධීන්ට අදාළ සම්භාවිතාවන් $P(A) = \frac{1}{5}, P(B) = \frac{1}{6}, P(A \cap C) = \frac{1}{20}, P(B \cap C) = \frac{1}{24}$ වනසේ අර්ථ දැක්වේ. P(C), P(B/C), P(C/B) හි අගයන් සොයා B හා C සිද්ධීන් ස්වායත්ත බව පෙන්වන්න.

බැගයක ආරම්භයේදී රතු බෝල 1 ක් හා නිල් බෝල 2 ක් අඩංගු ය. බැගයෙන් අහඹු ලෙස බෝලයක් ගෙන එහි වර්ණය නිරීක්ෂණය කර, තවත් එම වර්ණයේ බෝලයක් සමඟ එය ප්‍රතිස්ථාපනය කරනු ලැබේ. මේ ක්‍රියාවලිය තුන්වරක් කරනු ලැබූ විට, ලැබෙන ප්‍රතිඵල හා අදාළ සම්භාවිතාවන් නිරූපණය කරන රූක් සටහනක් අඳින්න.

ඒ නයින් හෝ අන් අයුරකින්,

(a) අඩුම වශයෙන් නිල් බෝලයක් ලැබීමේ

(b) හරියටම නිල් බෝල එකක් ලැබීමේ සම්භාවිතාවන් සොයන්න.

ඉවතට ගත් බෝල තුනම එකම වර්ණයෙන් බව දන්නේ නම් ඒවා රතු පැහැවීමේ සම්භාවිතාව සොයන්න.

10. (a) පහත වගුවෙන් වානේ කම්බි 100 ක, ආතන ප්‍රත්‍යාබල දැක්වේ.

ප්‍රත්‍යාබලය/100N/m ²	කම්බි ගණන
0-10	3
10-20	16
20-30	26
30-40	31
40-50	16
50-60	8

මධ්‍යස්ථය, මධ්‍යයනය හා සම්මත අපගමනය සොයන්න.

(b) පාඨාංක 48 ක මධ්‍යන්‍යය 7.50 ක් හා සම්මත අපගමනය 0.30 ක් බව සොයාගෙන ඇත. 6.8 හා 8.2 යන අතිරේක පාඨාංක දෙකක් පාඨාංක 48 සමග සැලකිල්ලට ගතයුතු නම් පාඨාංක 50 සඳහා වන මධ්‍යන්‍ය හා සම්මත අපගමනය සොයන්න.

11. (a) AB සැහැල්ලු අවිතන‍්‍ය තන්තුවක දිග 3G වන අතර එය එකම සිරස් රේඛාවේ එකිනෙකට 2a දුරින් වූ A හා B ලක්ෂ්‍ය දෙකකට ගැටගසා ඇත්තේ තන්තුව තුලින් ස්කන්ධය m වූ මුද්දක් ගමන් කරන සේය. B ට සිරස්ව ඉහළින් A පිහිටයි. R මුද්ද තන්තුවේ මධ්‍ය ලක්ෂ්‍යයේ තිබෙන විට, මුද්ද $\sqrt{5ga}$ වේගයෙන් තිරස් වෘත්තයක් ගෙවා යයි. තන්තුවේ AR හා RB කොටස්වල ආතති සොයන්න.

(b) ස්වභාවික දිග 1m හා ප්‍රත්‍යාස්ථා මාපාංකය 49N වූ තන්තුවක එක් කෙළවරකට 2kg ස්කන්ධය සහිත අංශුවක් අමුණා, තන්තුවේ අනෙක් කෙළවර O අවල ලක්ෂ්‍යයකට අමුණා තිබේ. අංශුව O හි මට්ටමට ගෙන නිදහස් කල විට, අංශුව පළමුවරට ක්ෂණික නිශ්චලතාවයට පත්වන විට, O හි සිට එම පිහිටුමට දුර සොයන්න.

සිදුවන චලිතයේ, දෝලනවල කාලාවර්තය සොයන්න.

12. AB ඒකාකාර දණ්ඩක දිග 2a හා ස්කන්ධය 3m වන අතර B කෙළවරෙහි ස්කන්ධය 2m වූ අංශුවක් අමුණා තිබේ.

A ලක්ෂ්‍ය හරහා යන සුමට අක්ෂයක් වටා දණ්ඩට සිරස්තලයක භ්‍රමණය විය හැක. දණ්ඩ A ට සිරස්ව ඉහළින් B කෙළවර පිහිටන සේ තබා, සෙමෙන් නිදහස් කරනු ලැබේ.

A හරහා වූ අක්ෂය වටා පද්ධතියේ අවස්ථිති ඝූර්ණය සොයන්න.

ශක්ති සංස්ථිති මූලධර්මය යෙදීමෙන්

$$\dot{\theta}^2 = \frac{79}{12a} (1 - \cos \theta) \text{ සමීකරණය පිහිටුවන්න.}$$

$\dot{\theta}$ යනු දණ්ඩ θ කෝණික විස්ථාපනයකදී දණ්ඩේ කෝණික ප්‍රවේගයයි.

අක්ෂය මත දණ්ඩේ ඇති කරන ප්‍රතික්‍රියාවේ විශාලත්වය දණ්ඩ $\frac{\pi}{3}$ රේඛය

කෝණයකින් භ්‍රමණය වී ඇතිවිට සොයන්න.

හිමිකම් ඇවිරිණි.

கிவங்கைத் திரைத் பல்கலைக்கழகம்

தினாழிவந்துட்பவியல் டிப்ளோமா - அடிப்படை மட்டம் 02



கிறித்திப்பரிட்சை - 2006

MPZ 2230 - கணிதம் - வினாத்தாள் II

105

தாலம் - ஸீனா (03) மணித்தியாலங்கள்.

திகதி : 19 - 03 - 2007

சேதரம் : 9.30 - 12.30 hrs.

ஒவ்வொரு பததியிலிருந்தும் ஆகக்குறைந்தது ஒரு வினாவையேயும்
தொறுபுத்து வினாத்தாள் ஆறு (06) வினாக்களுக்கு விடையளிக்க.
கணிப்பாண்களை பாவிக்கலாம்.
தையடங்கத்திதாலைபெசுகளை கணிப்பாண்களாக பாவிக்க
முடியாது.



பகுதி - A

01. $AB = AC$ ஆகலாறு ABC ஓர் திரசுமபங்கடுங்கிதாணியாகும்.
கிாடுகள் AB, BC யின் சமன்பாடுகள் $2x - y - 1 = 0$ உம் $x - 2y + 1 = 0$ உம் ஆகும்.
கிாடு AC ஆகலாறு கிாடு $2x + 11y = 0$ ங்கு சகாத்தரம்
தாண திறுது.
 A யின் ஆள்கூறு $(2, 3)$ தாணத்தரப்படிண் புள்ளி C யின்
ஆள்கூறுகளைக் காண்க.
டுங்கிதாணியின் பரப்பைக் காண்க.

02. (a) உட்டம் $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ உம்
கிாடுகிாடு $l \equiv px + qy + r = 0$ உம் ஒண்ணையாணறு
கிடைவடும் புள்ளிகள் A, B சமன்பாடு $S + \lambda l = 0$
கிணை திரப்தி லுய்யும். திங்கு λ ங் பரமாணம்.

AB ஐ வட்டமாகவுடைய வட்டம் S' கின் சமன்பாட்டைக் காண்க .

$S \equiv x^2 + y^2 - 6x + 2y - 17 = 0$ உம் $L \equiv x - y + 2 = 0$ உம்
 எண்ணின் வட்டம் S' உம் வட்டம் $x^2 + y^2 - 8x + 2y + 13 = 0$
 உம் வெளிப்புறமாகத் தொடும் எண்க்காட்டுக .

(b) வட்டம் S , $(2,0)$ கிணாடு லெவ்வதுடன் வட்டம்

$S' \equiv x^2 + y^2 - 1 = 0$ ஐ S' கின் வட்டத்தில் எதிரெதிர
 உள்ள புள்ளிகளில் வெட்டுகின்றது

S ஆனது வட்டம் $x^2 + y^2 - 4y - 5 = 0$ ஐ லெவ்விகாண்க
 கில் வெட்டும் எண்ணின் S கின் சமன்பாட்டைக் காண்க .

03. $y^2 = 4ax$ எண்கும் லெவ்வையியல் உள்ள புள்ளிகள்
 $(at_1^2, 2at_1)$, $(at_2^2, 2at_2)$ ஆகியவற்றை கிண்க்கும்
 தொண்ணின் சமன்பாட்டைக் காண்க .

லெவ்வையியல் குவியல் $S(a,0)$ கிணாடு லெவ்வும் தொண் PQ .

கொடு $x + a = 0$ லெவ்வ வலையப்பட்ட லெவ்வகத்தக்கள்

PM , QN எண்பன எக்கொட்டை லெவ்வைய M, N கில்

செத்திக்கின்றன. லெவ்வலவணவற்றை லெவ்வக .

(a) PN , QM எண்பன லெவ்வத்தியிலாடு லெவ்வகம்

(b) MN ஆனது குவியத்தில் லெவ்விகாண்க்கை எண்க்கும்

(c) லெவ்வையக்த P, Q லெவ்வ வலையப்படும் தொல்லககும்

SM, SN ஆகிய செத்தகொடு ^{கொடு} எண்க்கும் லெவ்வகத்தின்
 லெவ்வ லெவ்வ கொடு MN கில் கிண்க்கும்

(லெவ்வ : P, Q லெவ்வ வலையப்படும் தொல்லககள்
 கிணலெவ்வும் லெவ்வையக்த காண்க)

04. துயவுசெய்து பகுதி (a) அல்லது (b) யாண்டு விடையளிக்க.

(a) நீர்வளைவாய $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ன்ற புள்ளி $P(x_1, y_1)$ கில்
 ிசங்குத்தின் சமன்பாடு $\frac{a^2(x-x_1)}{x_1} = \frac{b^2(y-y_1)}{y_1}$ றன
 திமுயுத.

P யல் ஁ர்ற ிசங்குத்து x அச்சை Q வியும்
 y அச்சை R கியும் ிவட்டுகின்றது.

P யன் றல்லா திவைதருக்கும் $PQ : PR$ சூனத்து ஁ர்
 றாநிவி றனக்காட்டுக.

P யானது ஁ர் நீர்வளைவாயை விபரிப்பதைப்பீபல்,
 ிசங்குதம் $OQSR$ கின் ஁ச்சி S ஁ம் அடுத
 ஁ற்பத்தியைக் ிகாண்ட கின்஁னார் நீர்வளைவாயி
 -யை விபரிக்கும் றனக்காட்டுக.

(b) ிசங்குக்காண அதிபரவளைவாயி $xy = c^2$ கில் ஁ர்ற
 புள்ளிகள் $(ct, c/t), (cu, c/u)$ றன்பவற்றை
 கிணைக்கும் தூணின் சமன்பாட்டைக் காண்க.

அதிபரவளைவாயின் றாறும் தூண் சூனத்து புள்ளி $A(c, 2c)$
 யுறாடு ிசங்கின்றது.

தூணின் துடுப்புள்ளியின் ஁ழுக்கு $2xy = c(2x+y)$
 றனும் சமன்பாட்டால் துரப்பலாம் றனக்காட்டுக.

அத்தூண் அவ் றழுக்கு A யுறாடு ிசங்கு றனவும் காட்டுக.

A யல் அவ்றவாயுத்தின் ிசங்குத்தானது $xy = c^2$ றனும்
 அதிபரவளைவாயை B, C யல் ிவட்டுகின்றது.

$$BC^2 = \frac{85c^2}{4} \text{ றன திமுயுத.}$$

05. (a) $f(x) = 3 \cos^2 x + 2\sqrt{3} \sin x \cos x + \sin^2 x$, $x \in \mathbb{R}$,
என்க.

$f(x)$ ஐ $A + B \cos(2x - \alpha)$ எனும் வடிவில் எழுதிக்கொடுக்க.
மாறிலிகள் A, B, α ஆகியவற்றின் பெறுமானங்களைக்
கீழ்க்காண்க.

கிடைத்தால் $f(x)$ கின் உயர்வு, கிழிய பெறுமானங்களைக்
காண்க. அதற்கான x கின் பெறுமானங்களை $-2\pi \leq x \leq 2\pi$
ந்கிடைபல் காண்க.

(b) (i) பின்வரும் சமன்பாட்டைத் தீர்க்க

$$\cos 3x - \cos 5x = 2 \sin x$$

$$(ii) \tan^{-1}(1/2) + \tan^{-1}(1/3) = \tan^{-1}(1/4) + \tan^{-1}(3/5)$$

எனக்காட்டுக.

(c) முக்கோணி ABC யின் உழையையான சூழிடையி
யயன்படுத்தி

$$a^2 + b^2 + c^2 = \frac{abc}{R} [\cot A + \cot B + \cot C]$$

காட்டுக

பகுதி - B

06. தரப்போதலின் சிற்றத்தகல் w யுக் .

w ஂண்பது 1 கின் ஓர் சிங்கல் கனமேலம் ஂணின் w க் $\cos \theta + j \sin \theta$ கில் காண்க .

கிகிலாடாத்து $w + w^2 + 1 = 0$ ஂண்பதைப் ஓபயுக் .

பின்வநவணவற்றை நியுயுக் .

(i) $(1 - w + w^2)(1 + w - w^2) = 4$

(ii) $(1 - w)(1 - w^2)(1 - w^4)(1 - w^8) = 9$

(iii) $\frac{a + bw + cw^2}{b + cw + aw^2} = w$

(iv) $(1 + w - w^2)^3 - (1 - w + w^2)^3 = 0$

07. (a) $|z| = 1$ ஆதமாயு z ஆதாவது ஓர் சிங்கலெண்ணின் ஂணின் , ஆகண் வரிப்படத்தை உபயோகிகித்திர் அல்லது வேறு விதலாடலெயர் $1 \leq |z - 2| \leq 3$ ஂணயும் ஓர்சீசம் $\text{Arg } |z - 2| \geq 5\pi/6$, $\text{Arg } |z - 2| \leq -5\pi/6$ ஂணயும் நியுயுக் .

(b) A, B, C ஆகியவற்றின் தூணக்காவிசண் ஓணுபெய

$\underline{a} = 12\underline{i} + 8\underline{j} + \underline{k}$, $\underline{b} = 3\underline{i} + 2\underline{j} - 5\underline{k}$,

$\underline{c} = -3\underline{i} - 2\underline{j} + p\underline{k}$ ஆகும் .

(i) பின்வநம் சந்தார்ப்பநங்களில் p க்க் காண்க .

(α) \underline{c} யும் \overline{AB} யும் சநாந்தரமாத

(β) A, B, C ஓர் சிங்கலெண்ணின் அமையும் வண்ணலாத

(ii) p ஓர் மானி ஂணின் C யின் ஓடுக்க ஓர் சிங்கலெண்ணும் ஂணக்காட்டுக . சித்துடன் அக்கெட்டெண்ணும் வரிபெயுண கிந விசுக்காவிசணின் சமன்பாட்டெயும் காண்க .

11.(a) 3m நீளமுடைய கிளைசாண நீளா கிழை AB யின் ஓர் ஊண் புள்ளி A யையும் மறுஊண் புள்ளி B யையும் நிலைப்படுத்துப்பட்டுள்ளது. புள்ளி B யானது A யின் 2m தூரம் நிலைக்குத்தாக கீழேயுள்ளது. மதினியுடைய ஓர் சிறிய வண்டியும் R கிழையால் போர்க்கப்பட்டுள்ளது. R ஆனது கிழையின் நடுப்புள்ளியில் நிலைப்படுத்துப்பட்டுள்ளதுடன் $\sqrt{5}ga$ கதியுடன் ஓர் கிடைவட்டத்தில் கியங்குகின்றது. கிழையின் பகுதிகள் AR, BR கிள் கிழைவகணாக காண்க.

(b) கிளைசாண கிழை ஒன்றின் ஓர் ஊண்யால் 2 kg கிணியுடைய துணிக்கையொன்று கட்டப்பட்டுள்ளது. கிழையின் கியற்க நீளம் 1m. அதன் கீர்கண்மைமட்டு 49 N. அதன் மறுஊண் ஓர் நிலையான புள்ளி O கிள் போர்க்கப்பட்டுள்ளது. துணிக்கையானது புள்ளி O யில் பிடிக்கப்பட்டு விடுகப்படுகின்றது. புள்ளி O யில் கிடித்து, துணிக்கை கணநிலை ஆய்ந்த வரும் மட்டத்துக்கே காண்க. டிதாடர்ச்சியான கியக்கத்தின் கிழையு காலத்தையும் காண்க.

12. 2a நீளம், 3m கிணியுடைய ஓர் சீரான கோல் AB யில் 2m கிணியுடைய துணிக்கையானது அதன் ஊண் B யில் கிணைக்கப்பட்டுள்ளது. அக்கோல் ஆனது ஓர் நிலைக்குத்து தளத்தில் A யிளாடு ஊண்யும் ஒப்பமான அச்சுபற்றி சுழலவல்லது. அவ்வுலானது B யானது A யிற்கு நிலைக்குத்தாக மேலே கிடிக்கும் நிலையிலிடித்து சிறிது கிடப்பொக்கப்படுகின்றது. A யிளாடான அச்சுபற்றி உடலின் சுத்துவகிடிப்பத்திணைக் காண்க.

சுத்துக்கிடிப்பு வகிழைய பாவிக்கிதா அவ்வது வேறுவிதமாகவே

$$\theta^2 = \frac{7g}{12a} (1 - \cos \theta) \text{ என கிழையு}$$

கிடிடு $\theta - \theta$ கோண கிடப்பொக்கி θ யில் கோணகிடிக்கம் உடலானது $\pi/3$ ஆனரயணால் சுடிற்றப்படும் போது அச்சில் மறுகாக்கத்தின் படுணைக்க காண்க.