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THE OPEN UNIVERSITY OF SRI LANKA
COMMONWEALTH EXECUTIVE MASTER OF BUSINESS/ PUBLIC ADMINISTRATION
FINAL EXAMINATION -2015
MCP 1607 - QUANTITATIVE TECHNIQUES
DURATION: THREE (03) HOURS

DATE: 16 December 2015 **TIME: 09.30 am- 12.30 pm**

INSTRUCTIONS TO CANDIDATES.

a) **Answer any five (05) questions.**
b) **All questions carry equal marks.**
c) **Write your index number on every page.**
d) **Use of non-programmable calculators are allowed.**
e) **Necessary statistical tables and mathematical formulae given.**

(Q1) a. Find the differential coefficient of the following functions with respect to "x"

i) x^4+3x^2+7x+2 ii) $(x^3+3)(x^2+7)$ (8 marks)

b. If $y = x^3+2x^2+7$ find $\frac{d^2y}{dx^2}$ (4 marks)

c. The factory manager observes that the performance of his machines initially improves with age and there after gradually drops. If the relationship between daily output of machine and age of the machine is given by;

$R = 500 + 14x-x^2$ where "R" is the daily output and "X" is the age, find at what age the machine would give the best performance. (4 marks)

d. An agriculturist plans to cultivate a rectangular block of land. He has 1000 feet of fencing material to erect a fence along the perimeter of the block. He hopes to maximize the area of the block. Find the length and breadth of the block that would maximize area while keeping the perimeter as 1000 feet. (4 marks)

(Q2) a. A and B are two matrices defined as follows.

$$A = \begin{pmatrix} 7 & 9 & 3 \\ 5 & 8 & 4 \\ 6 & 7 & 9 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 4 & 3 & 1 \\ 3 & 6 & 1 \\ 2 & 5 & 6 \end{pmatrix}$$

Evaluate the following.

i) A + B (2 marks)

ii) A - B (2 marks)

iii) 3A - 2B (2 marks)

iv) A x B (vector multiplication) (4 marks)

b. Using Cramers rule in matrices algebra or inverse matrices method solve the following simulation equations.

$$\begin{aligned} 2x + 3y + z &= 11 \\ x + 2y + 3z &= 14 \\ 3x + y + z &= 8 \end{aligned}$$

(10 marks)

(Q3) a. The distribution of a sample of 800 vehicles in the Western Province by district vehicle registered and source of energy is given in the table bellow.

Source of Energy	District			Total
	Colombo	Gampaha	Kaluthara	
Petrol	300	150	100	550
Diesel	175	35	40	250
Total	**475**	**185**	**140**	**800**

If a vehicle is selected at random, find the following probabilities.

i) P (Petrol)

ii) P (Petrol ∩ Gampaha)

iii) P (Diesel ∪ Colombo)

iv) $P\left(\frac{\text{Petrol}}{\text{Gampaha}}\right)$

v) $P\left(\frac{\text{Gampaha}}{\text{Petrol}}\right)$

(8 marks)

b. A batch of thousand candidates who sat the examination had different methods of revising there work. Around 45% of them revised the full syllabus. 30% of them revised selected topics while 25% of them attempted past papers. Previous results suggest that among those who revise full syllabus, those who revise selected topics and those who attempt past papers, 40%, 70% and 50% pass the examination respectively. If a candidate for the examination is selected at randomly find the following probabilities.

i) Probability that candidate pass the examination.

ii) Given that the candidate pass the examination, what is the probability that he revised selected topics.

iii) Probability that candidate fail the examination.

iv) Given that the candidate fail the examination, what is the probability that he attempted past papers. (12 marks)

(Q4) a. i) Evaluate ${}^{n}C_{r}\, p^{r}\, q^{(n-r)}$ where $n = 5$, $r = 3$ and $P = 0.4$ (2 marks)

ii) An examination paper has multiple-choice questions. In each question the candidate has to select the correct answer out of four suggested answers. If a completely unprepared student may be assumed to guess each answer randomly, what is the probability that he guesses the correct answer? (3 marks)

iii) In the above example, if the question paper has five such multiple-choice questions what is the probability that he gets three or more questions correct? (4 marks)

b. i) Evaluate $e^{-a}\frac{a^{x}}{x!}$ where $a = 2$ and $x = 3$ $e = 2.71$ (2 marks)

ii) The annual number of earthquakes occurring at least 2.5 on the richter scale follows a poisson distribution with mean 3. What is the probability that there will be at least two such earthquakes next year. (4 marks)

c. The life time of a certain type of light bulb is approximately normally distributed with mean 1000 hours and standard deviation 200 hours. In a consignment of 900,000 such bulbs how many bulbs will approximately burn less than 700 hours. (5 marks)

(Q5) a. Compare and contrast random and non-random sampling. (6 marks)

b. A sample of 36 packets of tea were weighed and the weight of each packet recorded. It was observed that the mean weight of a packet of tea was 245 grams with standard deviation 22 grams. Assuming that the weight is normally distributed, develop a 95% confidence interval estimate for the weight of a packet of tea. (7 marks)

c. Medical science states that the content of compound "C" in a certain tablet should be 30mg. The manufacturer of this tablet claims that his tablet contains exactly 30mg of compound "C". To test this, a sample of 40 tablets were weighed and weight of each tablet recorded. It was observed that the mean weight of compound "C" in a tablet was 34 with standard deviation 6, assuming that the weight of tablet is normally distributed, carry out a statistical test at 5% level of significance to test the claim made by the manufacturer (7 marks)

(Q6) An investigation is conducted to study the gasoline millage in auto mobiles. 10 properly tuned cars were used in the study. Each car is driven for 1000 kilometers and average number of kilometers per liter is obtained. In the table below "Y" represents the average number of kilometers per liter and "X" represents the weight of the car in metric tons. The study is to find the impact of weight on average kilometers per liter.

Car Number	1	2	3	4	5	6	7	8	9	10
Y	17.9	16.5	16.4	16.8	18.8	15.5	17.5	16.4	15.9	18.3
X	1.35	1.9	1.7	1.8	1.3	2.05	1.6	1.8	1.85	1.4

$\varepsilon x = 16.75 \quad \varepsilon y = 170 \quad \varepsilon x^2 = 28.64 \quad \varepsilon y^2 = 2900 \quad \varepsilon xy = 282.4$

(i) Calculate the correlation coefficient between weight and average kilometers per liter. (3 marks)

ii) Evaluate the line of regression $y = a + bx$ (3 marks)

iii) Estimate the average kilometers per liter when weight is 2.5 (mt) (2 marks)

iv) What is the residual of observation where $x = 1.6$ (2 marks)

v) Evaluate the sum of squares error giving by "SSE" (2 marks)

vi) Calculate the coefficient of determination (R^2) and interpret the result. (2 marks)

vii) Calculate the standard error of "b" coefficient given by Sb1 (3 marks)

viii) Develop a 95% confidence interval estimate for the "b" coefficient (3 marks)

Mathematical Formula

i) Correlation coefficient

$$r = \frac{\varepsilon xy - \frac{(\varepsilon x)(\varepsilon y)}{n}}{\sqrt{\left[\varepsilon x^2 - \frac{(\varepsilon x)^2}{n}\right]\left[\varepsilon y^2 - \frac{(\varepsilon y)^2}{n}\right]}}$$

ii) Line of regression $y = a + bx$

$$b = \frac{n\varepsilon xy - (\varepsilon x)(\varepsilon y)}{n\varepsilon x^2 - (\varepsilon x)^2} \qquad a = \frac{\varepsilon y}{n} - \frac{b\,\varepsilon x}{n}$$

iii) $SSE = \varepsilon y^2 - a\varepsilon y - b\varepsilon xy$

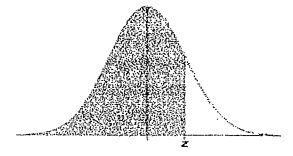
iv) Coefficient of determination $= r^2$

v) $Syx = \sqrt{\frac{SSE}{n-2}}$

vi) $Sb1 = \frac{Syx}{\sqrt{SSX}}$ where $SSX = \varepsilon x^2 - \frac{(\varepsilon x)^2}{n}$

vii) Standard error $= \frac{S}{\sqrt{n}}$

Standard Normal Cumulative Probability Table



Cumulative probabilities for POSITIVE z-values are shown in the following table:

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998