THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF INDUSTRIAL STUDIES / BACHELOR OF TECHNOLOGY

TTZ5244 – QUANTITATIVE TECHNIQUES

FINAL EXAMINATION 2006/2007

DURATION – THREE HOURS

Date 6th April 2007 TIME: 0930 -1230 hrs.

Answer 05 Questions including question 01 which is compulsory. All questions carry 20 marks amounting to total of 100.

(1) (i) Solve the following equations **(02 marks)**

(a) $5^{x+2} = 125^{2-x}$ (b) $2^{x+2} + 5 = 261$

(ii) Explain the following expressions with x as the subject .

(i) $y = Z(1 + r)^x$ (ii) $(2 + y)\,2^x = 4$

(iii) $y = e^{x^2}$ (iv) $a^x = 50$ $(a>0)$ **(02 marks)**

(iii) If f(x) = x–1/x+1, what is f(0), f(1) and f(-1) **(02 marks)**

(iv) Calculate the gradient of the following straight lines

(a) $y = 100 - 0.5x$ **(02 marks)**

(b) $ax + by = C$

(v) Find the stationary points of the following function. **(02 marks)**

$y = 3x^3 - 3x - 5$

(vi) Determine the second derivative of the following function

$y = (x - 1)\dot{x}$ **(02 marks)**

(vii) Give an example for a diagonal matrix. **(02 marks)**

Consider following matrices for questions (viii) and (ix)

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}_{2x2} \quad B = \begin{pmatrix} 2 & 4 \\ 5 & -2 \end{pmatrix}_{2x2} \quad \text{and} \quad C = \begin{pmatrix} -1 & 2 \\ -3 & 2 \end{pmatrix}_{2x2}$$

(viii) Determine the value of A + B + C **(02 marks)**

(ix) Determine the value of $A^T + B^T$ **(02 marks)**

(x) Find the determinant of the matrix A, if **(02 marks)**

$$A = \begin{pmatrix} 0 & 2 & 1 \\ -2 & 0 & 3 \\ -1 & -3 & 0 \end{pmatrix}_{3x3}$$

(2) (i) What are the two important parameters in a straight line graph?
(02 marks)

(ii) Describe how linear equations are used in modelling. **(06 marks)**

(iii) Two products A & B are produced by a manufacturer. Each product is processed on two machines M_1 and M_2. Product A requires one minute of processing time on M_1 and two minutes on M_2, while B requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 7 ½ hrs, while machine M_2 is available for 10 hrs during any working day. Take number of units to be produced on A and B are variables.

Write the constraints of this problem mathematically **(02 marks)**

Plot the graph of the two variables mentioned above and mark the feasible region in it. **(10 marks)**

(03) (a) Differentiate the following function with respect to X

(i) $Y = (X-2)(X^2 + 2X + 4)$

(ii) $Y = 5 \log_e (X^2 + 2X + 1)$

(iii) $Y = (8X + 1)^2$

(iv) $Y = e^{(5X+1)}$ **(16 Marks)**

(b) If $X^2 + 2X + 3y^2 = 2$, determine the value of $\frac{dy}{dx}$ at $y = 1$

(04 Marks)

(04) A company incurs a fixed production cost of Rs. 1280 and variable cost of Rs 80 per unit output It's demand function is $P = 100 - Q/20$, where P is the Unit price and Q is the number of units of demand.

(i) Write down an equation for the total cost of production. **(02 Marks)**

(ii) Express revenue as a function of Q

(Revenue 'R' is given by R= PQ) **(02 Marks)**

(iii) Express the total profits as a function of Q

(Profit = revenue – total cost) **(04 Marks)**

(iv) Sketch the graph of profit as a function of Q. **(08 Marks)**

(v) How many units should be produced in order to maximise the profit.

(04 Marks)

(05) (a) Find the inverse of the matrix.

$$A = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 5 & 1 \\ 2 & -1 & 4 \end{pmatrix}_{3x3}$$

(05 Marks)

(b) Using the result obtained in 5(a) solve following equations.

$$\begin{aligned} X - 2Y + Z &= 6 \\ X + 5Y + Z &= -1 \\ 2X - Y - 4Z &= 1 \end{aligned}$$

(15 Marks)

(06) A person wants to decide the constituents of a diet which will fulfil his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in the table.

Food type	YIELD per unit			Cost per Unit
	Proteins	Fats	Carbohydrate	
A	3	2	6	40
B	4	2	4	40
C	8	7	7	85
D	6	5	4	65
Minimum requirement	800	200	700	

(i) What are the variable in this problem? **(04 marks)**

(ii) Write the objective of this problem, mathematically **(08 marks)**

(iii) Write the constraints of the problem, mathematically **(08 marks)**

(07) A soft toy factory has 1000 man-hours available each week for production and can spend up to Rs.700 a week on new materials. They produce only 2 styles A and B. It takes 2 man-hours to manufacture A and 1 man hour to manufacture B. It costs Rs 1 in raw materials to make either A or B.
The company makes a profit of Rs. 2 on each of the product A and Rs. 1.50 on each of the product B.

(a) Name the variables in this problem. **(02 marks)**

(b) What are the constraints of the problem? **(02 marks)**

(c) What is the objective of the problem? **(02 marks)**

(d) Solve the formatted programme **graphically** to determine the number of products A and B to be manufactured to get maximum profit? **(10 marks)**

(e) Calculate the maximum profit. **(04 marks)**

(08) Two types of product x and y are produced in a factory. Because of the raw material restrictions, not more than 400 tonnes of types x and 300 tonnes of type y can be produced in a week. There are 160 production hours in a week. It requires 0.2hrs and 0.4hrs to produce one tonne of product x and y respectively. The profit of Rs. 20 and Rs. 30 per tonne for product x and y respectively.

(a) Formulate the problem as a linear programme model. **(05 marks)**

(b) Determine how many of each type to be manufactured in order to maximise the profits using **Simplex Method.** **(15 marks)**