039

THE OPEN UNIVERSITY OF SRI LANKA
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING
ECX4242/ECD2215 – CONTROL SYSTEMS
FINAL EXAMINATION– 2006/2007 **CLOSED BOOK**

DATE: 7th April 2007 **0930 hrs – 1230 hrs**

Answer the question 1 and any other 4 questions. All questions carry equal marks.

Q1) (a)

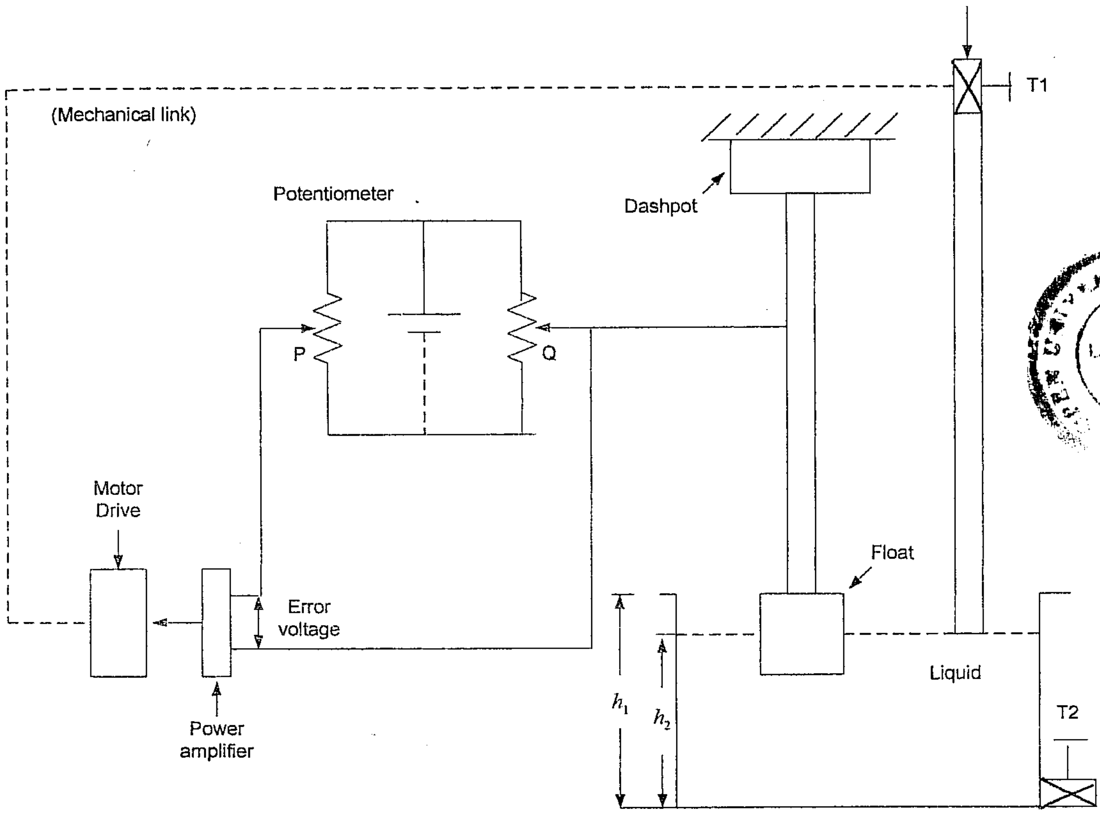


Figure Q1(a)

Figure Q1(a) shows a closed loop control system which is used to maintain the liquid level of a tank. Here, our objective is to keep the liquid level at the height of h_1. The existing liquid level is h_2 and T_1, T_2 are the inlet valve and outlet valve respectively.

(i) By studying the given schematic diagram, explain the operation of this liquid level control system.

(ii) Draw a complete block diagram, identifying each block and write down the functions of each unit.

(b) Figure Q1(b)shows a spring mass dashpot system mounted on a massless cart. Here, x(t) is the displacement of the cart and is the input to the system and the displacement y(t) of the mass is the output. At t=0, the cart is moving at a constant speed. In this system, m denotes the mass, b denotes the viscous friction coefficient and k denotes the spring constant.
Obtain the necessary differential equations for the dynamics of this system by assuming that the cart is standing still for t<0 and the spring-mass-dashpot system on the cart is also standing still for t<0.

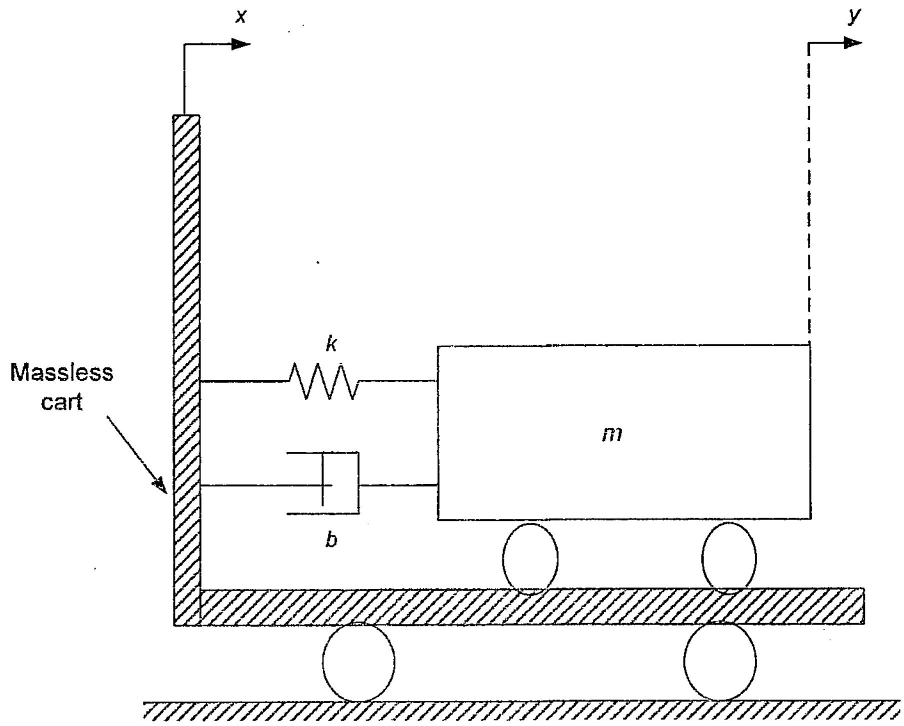


Figure Q1(b)

Q2) (i) Using the differential equations obtained in Q1(b), find the transfer function of the system when $b = 9$, $k = 20$ and $m = 1$.

(ii) Find the output $y(t)$, when

i. The input is a unit step. i.e. $x(t) = u(t)$

ii. The input is a ramp input. i.e. $x(t) = tu(t)$.

(iii) Obtain the steady state value for each case.

Q3)

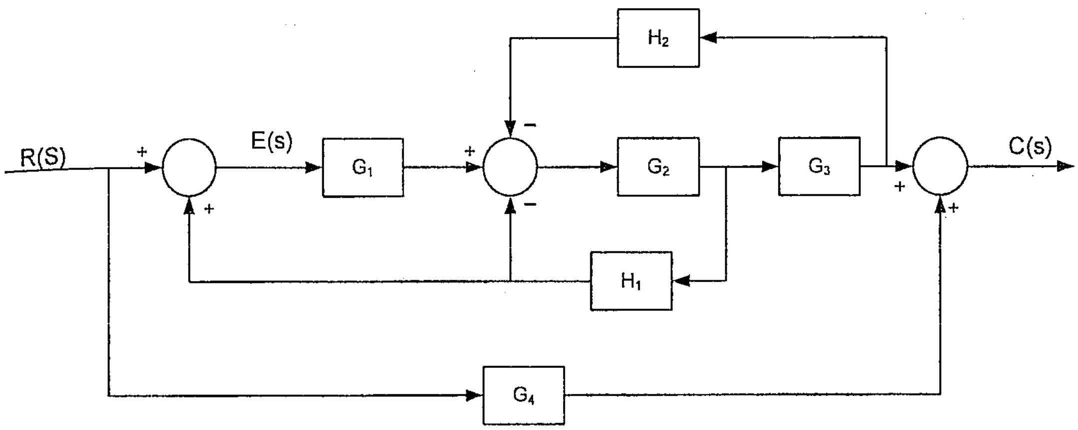


Figure Q3

(a) Draw the signal flow graph for the block diagram given in figure Q3.

(b) Use Mason's Gain Formula to obtain the transfer function C(s)/R(s) of the system. (Clearly show all steps.)

(c) Hence or otherwise find E(s)/R(s).

Q4) (a) Define the following terms regarding a 2nd order transfer function

i. Percent Overshoot
ii. Settling time
iii. Rise time and
iv. Peak time

(b) System performance specifications are often given in the time domain. Consider the following specifications:

Percent Overshoot <= 16%
Settling Time (2%) <= 2 s
Rise Time <= 0.6 s

Based on these specifications, answer the following questions.

(i) Sketch the lines defining each of these specifications in the s-plane. Shade the area of the graph that meets all three specifications. Be sure to indicate which line(s) correspond to which specification.

(ii) Determine a 2nd order transfer function that meets the first two specifications exactly (i.e. %OS = 16% and Settling Time = 2s).

(iii) Roughly sketch the step response for the transfer function found in (b)(ii) showing the rise time, settling time and percent overshoot on your sketch.

Note : You can use the data in Table A to answer this question.

Q5) A unity negative feedback control system has the open loop transfer function

$$G(s)H(s) = \frac{K}{s^2(s+3)} \quad \text{where } K \text{ is a positive constant.}$$

(i) Determine the characteristic equation of the system.

(ii) Sketch the Root Locus as K increases from 0 to ∞, stating the locations and directions of significant features of the diagram.

(iii) The system is modified so that

$$G(s)H(s) = \frac{K}{s(s+3)(s+a)} \quad \text{where } a \text{ is a positive constant.}$$

Now determine the modified characteristic equation and describe the effect on the Root Locus as the value of a is increased from 0 to 3. Detailed work is not necessary for this part. Just show how the Root Locus changes by using a simple sketch.

Q6) (a) Briefly explain the following terms related to a control system.

i) Frequency response

ii) Relative stability

iii) Bandwidth

(b) Consider the open loop transfer function of a unity negative feedback closed loop control system as

$$G(s) = \frac{100}{s(s+2)(s+3)}$$

(i) Draw the asymptotic Bode Plot of the system.

(ii) Obtain the phase margin and the gain margin of the system and comment on the stability of the system.

Q7) Plot the Nyquist diagram for a unity negative feedback control system with open loop transfer function :

$$G(s)H(s) = \frac{K}{s^2(s+3)} \quad \text{where K is a positive constant.}$$

Hence determine the range of K for which the system is stable.
Find the gain margin and phase crossover frequency of the system.

Q8) The open loop frequency response of a unity feedback control system is given in Table Q8. A series lead compensator, having a transfer function (1+0.5s), is to be inserted in the forward path of the system to improve its performance.

ω (rad/s)	1	2	5	10	20	30
Gain (dB)	26	20	4	-10	-20	-24
Phase (deg)	-120	-140	-164	-176	-184	-190

Table Q8

(i) Construct the straight line approximations to the Bode plots of gain (dB) and phase for the compensator.

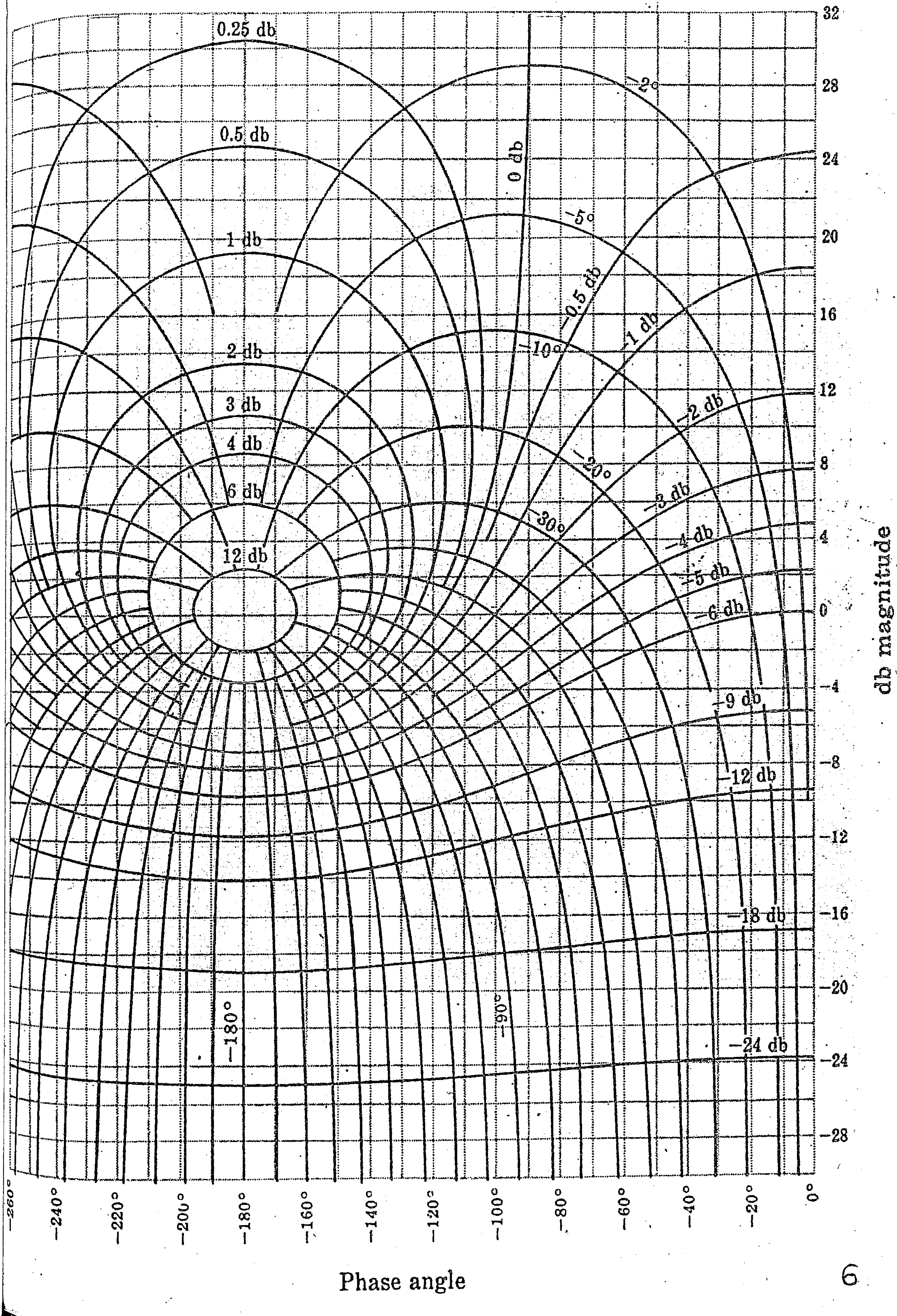
(ii) Plot the Nichol's Chart for the compensated system and for the uncompensated system.

(iii) Estimate the 3dB bandwidth, in rad/s, of the compensated closed loop control system. Compare this with the uncompensated system.

Selected equations of Time Response

$G(s) = \dfrac{\omega_n^2}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$	$\sigma = \zeta\omega_n$ $\omega_d = \omega_n\sqrt{1-\zeta^2}$
$t_r \cong \dfrac{1.8}{\omega_n}$	$t_s \cong \dfrac{4}{\sigma}$
$\zeta = \dfrac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}}$	$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}, 0 \le \zeta < 1$ and $M_p = \%OS/100$
$T_p = \dfrac{\pi}{\omega_n\sqrt{1-\varsigma^2}} = \dfrac{\pi}{\omega_d}$	$\%OS \cong \begin{cases} 5\%, \zeta = 0.7 \\ 16\%, \zeta = 0.5 \\ 20\%, \zeta = 0.45 \end{cases}$

Table A


0.25 db
0.5 db
1 db
2 db
3 db
4 db
6 db
12 db
0 db
−0.5 db
−1 db
−2 db
−3 db
−4 db
−5 db
−6 db
−9 db
−12 db
−18 db
−24 db
−2°
−5°
−10°
−20°
−30°
−90°
−180°
32
28
24
20
16
12
8
4
0
−4
−8
−12
−16
−20
−24
−28
db magnitude
−260°
−240°
−220°
−200°
−180°
−160°
−140°
−120°
−100°
−80°
−60°
−40°
−20°
0°
Phase angle