85

THE OPEN UNIVERSITY OF SRI LANKA

COMMONWEALTH EXEUTIVE MASTER OF BUSIENSS/PUBLIC ADMINISTRATION

FINAL EXAMINATION 2018

MCP1607 /MSP9407 – QUANTITATIVE TECHNIQUES

DURATION: THREE (03) HOURS

DATE : 18th July 2018	TIME: 9.30 am to 12.30 pm

INSTRUCTION TO CANDIDATES

a) **Answer any five (05) questions only.**
b) **Each question carries 20 marks.**
c) **Write your index number on every page.**
d) **Use of non-programmable calculator is allowed.**
e) **Necessary statistical tables and formulae annexed.**

Q1 a) The electricity board runs two plans namely plant (A) and plant (B) to generate electricity. They consume both diesel and petrol. The quantity of fuel consumed by both plants for the month of May is explained in the table below. The unit price of diesel and petrol is Rs.130/= and Rs.220/= per liter respectively. The expenditure for fuel to run both plants in the month of May is "x" for diesel and "y" for petrol. (expenditure measured in Rs."000"). Develop this relationship as a matrix equation.

PLANT	QUANTITY OF FUEL CONSUMED (IN "000" LITERS)	
	DIESEL	PETROL
A	15	4
B	9	3

b) A and B are two matrices defined as follows.

$$A = \begin{bmatrix} 7 & 5 & 9 \\ 12 & 8 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 & 5 \\ 6 & 4 & 3 \end{bmatrix}$$

Evaluate the following

(i) A + B (ii) A – B
(ii) 2 x A (iv) 3A + 2B

c) Use Cramer's rule to solve the following simultaneous equations.

$$2x + y + 3z = 13$$
$$4x + 3y + 2z = 16$$
$$x + y + z = 6$$

Q2. (a) Find the "Differential coefficient of the following functions with respect to "x".

(i) $2x^2 + 3x + 7$ (ii) $(x^2 + 4)(x + 3)$

(iii) $\frac{3}{x^2} + 5$ (iv) $\sqrt{x}$

(b) If $y = x^3 + 7x^2 + 4x + 3$ Find $\frac{d^2y}{dx^2}$

(c) The factory manager observes that the inventory cost would depend on the size of order that is placed each time. The cost is high, when order size is high as well as when order size is low. The manager intends to find the order size that would minimize inventory cost. The following equation gives the relationship between inventory cost and order size.

$$y = \frac{5400}{x} + 6x$$

In this equation "y" is inventory cost and "x" is the order size.

Find the optimal order size that would minimize inventory cost.

Q3 (a) Breakdown of 800 students by gender and performance at the examination is explained in the table below. Performance is classified as "Good", "Average" and "Poor".

GENDER	PERFORMANCE AT EXAMINATION			TOTAL
	GOOD	AVERAGE	POOR	
MALE	80	120	100	300
FEMALE	325	100	75	500
TOTAL	405	220	175	800

A student is selected at random. Find the following probabilities.

(i) P(MALE) (ii) P(GOOD) (iii) P(MALE ∩ GOOD)

(iv) $P\left(\frac{FEMALE}{GOOD}\right)$ (V) $P\left(\frac{GOOD}{FEMALE}\right)$

(b) On a certain highway the police have three strategies to control traffic. They are

(i) Employ full police force.
(ii) Employ half of the police force.
(ii) Employ just one police officer

It is observed that out of past 100 days, on 30 days police has used full force. On 50 days they have used half of the police force and on 20 days they have employed one police officer.

The probability that a motorist escape police is 0.3 when the full force is employed, it is 0.5 when half of the police force is employed and 0.8 when only one police officer is employed.

(i) What is the probability that the motorist escape police.
(ii) Given that the motorist escape police, what is the probability that police used full force.
(iii) Given that the motorist did not escape police, what is the probability that police employed just one officer.

Q4. (a) (i) Evaluate ${}^{n}C_{r}\ p^{r}\ q^{(n-r)}$

When $n = 5,\ r = 3$ and $p = 0.2$

(ii) Bulbs are sold in boxes and each box contains six bulbs. The probability that a bulb is damaged is 0.3. What is the probability that two out of the six bulbs are damaged .

(b) (i) Evaluate $e^{-a}\ \frac{a^{x}}{x!}$ When $a = 2,\ x = 3$ and $e = 2.71$

(ii) At a Barber shop on the average two customers arrive every hour. What is the probability that three customers will arrive the next hour.

(c) (i) The life span of a battery is normally distributed with mean 30 months and standard deviation 5 months. What percentage of batteries will have a life span of less than 35 months.

Q5 (a) (i) Why do people resort to sampling instead of studying the full population? Explain.

(ii) Discuss briefly the importance of evaluating sampling error in a study based on a sample.

(iii) In a study to estimate household monthly income, a sample of 225 households were surveyed and it was observed that the mean monthly income of households was Rs.18000/= and the standard deviation was Rs. 3000/=.

a) Develop a 95% confidence interval estimate for monthly income.
b) Develop a 80% confidence interval estimate for monthly income.
c) By how much should the sample size be increased to reduce the width of the confidence interval by half.

Q6 (a) It is believed that cost of maintenance and age of a machine are related. In the following table "x" represent age and "y" represent annual cost of maintenance.

x	y	x^2	y^2	xy
7	6	49	36	42
4	2	16	4	8
9	5	81	25	45
12	9	144	81	108
8	7	64	49	56
Total 40	**29**	**354**	**195**	**259**

The terms x^2, y^2, xy and their summations have been calculated for your convenience.

(a) Calculate the correlation coefficient between age and cost of maintenance.
(b) Evaluate the line of regression of the form y = a + bx.
(c) Predict the cost of maintenance "y" when age "x" is 7 years.
(d) Calculate the residual for each of the observation of "y".
(e) Calculate the mean of the residuals and comment on the result.

Q7 Write short notes on the following.
(a) Residual Analysis.
(b) Time series
(c) Sampling
(d) Probability

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MATHEMATICAL FORMULAE

CORELATION COEFFICENT $$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)\left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$

LINE OF REGRESSION $Y = a + bx$

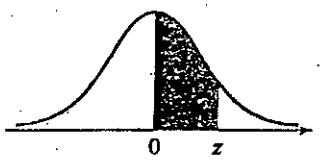
$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

95% CONFIDENCE INTERVAL:- Mean $\pm$ 1.96 $\left(\frac{SD}{\sqrt{n}}\right)$

80% CONFIDENCE INTERVAL :- Mean $\pm$ 1.28 $\left(\frac{SD}{\sqrt{n}}\right)$

Normal Curve Areas

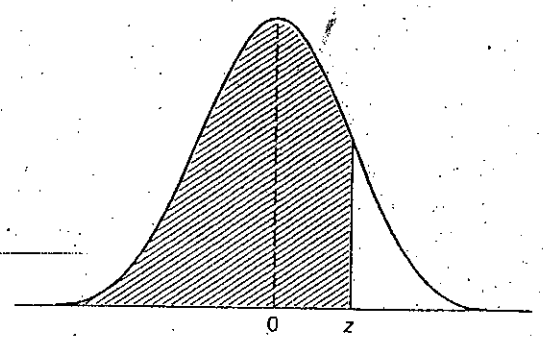


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

Table A2. Values of z, the standard normal variable, from 0·0 by steps of 0·01 to 3·9, showing the cumulative probability up to z.
(Probability correct to 4 decimal places).

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7704	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·9015
·3	·9032	·9049	·9066	·9082	·9099	·9115	·9131	·9147	·9162	·9177
·4	·9192	·9207	·9222	·9236	·9251	·9265	·9279	·9292	·9306	·9319
·5	·9332	·9345	·9357	·9370	·9382	·9394	·9406	·9418	·9429	·9441
·6	·9452	·9463	·9474	·9484	·9495	·9505	·9515	·9525	·9535	·9545
·7	·9554	·9564	·9573	·9582	·9591	·9599	·9608	·9616	·9625	·9633
·8	·9641	·9649	·9656	·9664	·9671	·9678	·9686	·9693	·9699	·9706
·9	·9713	·9719	·9726	·9732	·9738	·9744	·9750	·9756	·9761	·9767
2·0	·9772	·9778	·9783	·9788	·9793	·9798	·9803	·9808	·9812	·9817
·1	·9821	·9826	·9830	·9834	·9838	·9842	·9846	·9850	·9854	·9857
·2	·9861	·9864	·9868	·9871	·9875	·9878	·9881	·9884	·9887	·9890
·3	·9893	·9896	·9898	·9901	·9904	·9906	·9909	·9911	·9913	·9916
·4	·9918	·9920	·9922	·9925	·9927	·9929	·9931	·9932	·9934	·9936
·5	·9938	·9940	·9941	·9943	·9945	·9946	·9948	·9949	·9951	·9952
·6	·9953	·9955	·9956	·9957	·9959	·9960	·9961	·9962	·9963	·9964
·7	·9965	·9966	·9967	·9968	·9969	·9970	·9971	·9972	·9973	·9974
·8	·9974	·9975	·9976	·9977	·9977	·9978	·9979	·9979	·9980	·9981
·9	·9981	·9982	·9982	·9983	·9984	·9984	·9985	·9985	·9986	·9986
3·0	·9987	·9987	·9987	·9988	·9988	·9989	·9989	·9989	·9990	·9990
·1	·9990	·9991	·9991	·9991	·9992	·9992	·9992	·9992	·9993	·9993
·2	·9993	·9993	·9994	·9994	·9994	·9994	·9994	·9995	·9995	·9995
·3	·9995	·9995	·9995	·9996	·9996	·9996	·9996	·9996	·9996	·9997
·4	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9998
·5	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998
·6	·9998	·9998	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·7	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·8	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·9	1·0000									



The curve is $N(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1{\cdot}96$ the shaded area is 0·9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.