

THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY- FOUNDATION (LEVEL 01)
FINAL EXAMINATION 2006
MPZ 1331/MPF 1331 – APPLIED MATHEMATICS II



DURATION – THREE (03) HOURS

DATE : 11th March 2007

TIME: 0930 – 1230 hrs.

ANSWER (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS. YOU CAN'T USE MOBILE PHONES AS CALCULATORS.

SECTION - A

01. A particle moving in a straight line OP with uniform retardation of, leaves point O at time $t = 0$ with initial velocity U. It comes to instantaneous rest at P. On its way to P the particle passes points A,B,C at times $t = t_0, 2t_0, 4t_0$ respectively after leaving O. Where $AB = BC = a$.

By applying the equation of motion,

- a) Show that

i. $u = \frac{3a}{2t_0}$

ii. $f = \frac{a}{3t_0^2}$

- b) Find interms of a, the length of CP and the length of OA.
c) Find the total time for the journey OP.

Draw the velocity, time graph and acceleration time graph.

02. A boat moving at 8km/h relative to the water travels from a point A to a point B 10 km distance whose bearing from A is 150° . It then travels to a point C, 10km from B and due west of B. If there is a current of constant speed 4 km/h from north to south. Find the two courses to be set, and prove that the total time taken to reach C approximately 2 hours 20 minutes.

03. A particle thrown upwards from the top of a vertical cliff 56m high falls into the sea 4 seconds later 32m from the foot the cliff. Find the speed and direction of projection. The particle moves in a vertical plane, perpendicular to the cliff. A second particle is thrown at the same time, in the same vertical plane at the same speed and at the same angle to the horizontal, but downwards.

Find how long it will take to reach the sea and the distance between the points of entry of the particles into the water.

04. a) Three smooth spheres A,B,C of equal radii and masses, m , $2m$ and $4m$ respectively, are free to move along a straight horizontal groove with B between A and C. When any two spheres collide the impact is direct and the coefficient of restitutions is e . Spheres B and C are initially at rest and sphere A is projected towards sphere B with speed U . Show that the velocities of A and B after 1st impact are $\left(\frac{1-2e}{3}\right)u$ and $B(1+e)\frac{u}{3}$ respectively. Find the velocities of B and C after the second impact.
- b) Water is pumped at the rate of $1.2 \text{ m}^3/\text{min}$ from a large tank on the ground, up to a point 8m above the level of the water in the tank. It emerges as a horizontal jet from a pipe with a cross-section of 0.005m^2 . If the efficiency of the motor is 60%, find the energy supplied to the pump per second.

SECTION - B

05. force of magnitudes 2,1,2,3,2 and 1N act along the sides AB,BC,CD,ED,EF and AF respectively of a regular hexagon of side $2a$ in the directions indicated by the letters. Prove that this system of forces can be reduced to a single force of magnitude $2\sqrt{3} \text{ N}$ acting along AC together with a couple. Find the magnitude of the couple. Show that the system can be reduced to a single force with out a couple. If the line of action of this force cuts FA produced at X. Calculate length of AX.
06. A non uniform rigid beam PQ of length $3d$ and weight $8W$, rests on supports A and B at the same level. Where $PA=AB=BQ=d$; when a load W hung from P, the beam is on the point of tilting about A. Find the distance of the centre of gravity of the beam from P. When an additional load of weight W_1 is hung from Q, the forces exerted on the supports at A and B are equal. Find the value of W_1 .

If a couple of moment M and acting in the vertical plane through PQ , is now applied to the loaded beam, the reaction at A increased in the ratio. $3:2$ show that $M = 3wd$.

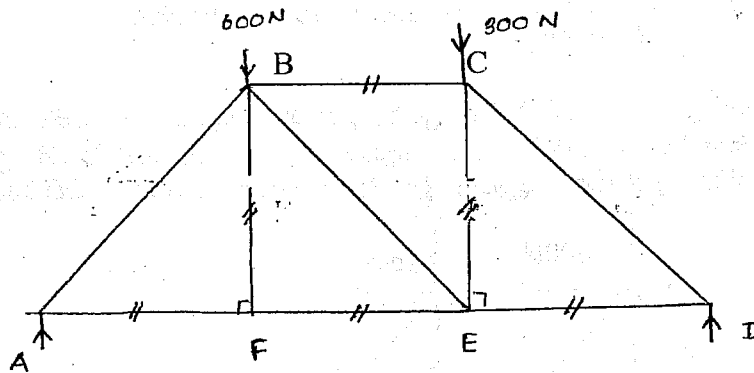
07. a) A ladder whose centre of gravity is at a point of trisection leans in a vertical plane with one end on rough horizontal ground and the other end against a rough vertical wall such that the centre of gravity of the ladder is near to the wall. If the coefficient of friction at each point of contact is μ and the ladder is on the point of slipping when it is incline at angle θ to the horizontal. Prove that

$$\cot \theta = \frac{3\mu}{2 - \mu^2}.$$

- b) A heavy uniform sphere rests on two smooth inclined planes whose inclinations to the horizontal are α and β . If α is given, and the reaction on this plane is half of the weight of the sphere prove that β must be

$$\tan^{-1} \left(\frac{\sin \alpha}{2 - \cos \alpha} \right).$$

08. The framework $ABCDEF$ is smoothly supported at A and D and carries weights of 600N and 300N at B and C respectively. Find the reactions at A and D . By using graphical method find the forces in the rods, stating which rods are in compression.



SECTION - C

09. A plane lamina consist of a circular disc of centre O and radius $2a$, from which a circular portion of centre C and radius a has been removed. $OC=a$. The lamina is completely immersed in homogeneous liquid with its plane vertical and C vertically bellow O and it is found that the centre of pressure of the lamina is at O. Show that the depth O below the free surface is $\frac{11a}{4}$.

You can assume that the centre of pressure of a vertical circular are of radius r immersed with its centre at a depth h is $\frac{r^2}{4h}$.

10. a) An alloy of two metals, whose specific gravities are 11.5 and 75. weights 4.5kg in air and 40 kg in water. Find the weight of each metal and the ratio of their values.
- b) 100g of a liquid of specific gravity 0.80 mixed with 80cm^3 of a liquid of specific gravity 0.96. No change of volume taking place, Find the specific gravity of the mixture.

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