MPZ 3230 – Engineering Mathematics I

Assignment No. 04 –Academic Year - 2008

Answer all questions

(1) (a) Evaluate $\int_0^1 \frac{1}{1+x}dx$ using Trapezoidal rule taking $h = \frac{1}{4}$

(b) Find the approximate value of $\log_e 5$ to four decimal places by calculating

$\int_0^5 \frac{1}{4x+5}dx$ (Divide the range in to 10 equal parts) using Simpson $\frac{1}{3}$ Rule

(2).

(i). Given, $\sin 55^0 = 0.8192, \sin 60^0 = 0.8660,\ \sin 45^0 = 0.7071, \sin 50^0 = 0.7660$

Find sin 52^0, using Newton forward interpolation formula.

(ii).

X	0.1	0.2	0.3	0.4	0.5	0.6
F(x)	2.68	3.04	3.38	3.68	3.96	4.21

Find f(0.7) approximately using suitable form.

(iii). Obtain the values of t when A=85, from the following table, using the Lagrange's methods,

X	2	5	8	14
F(x)	94.8	87.9	81.3	68.7

(iv). Apply Lagrange's formula inversely to obtain the root of the equation f(x) =0, given that,
$f(30) = -30,\ \ f(34) = -13,\ \ f(38) = 3\ and\ f(42) = 18$

(3) A box X contains two white balls, one blue ball and three red balls: a box Y contains four white balls, two blue balls and no red balls. An ordinary die is thrown once and, if the score is a 1 or a 6.then a ball is drawn at random from box X, otherwise a ball is drawn from box Y.

(a) Draw a tree diagram to represent this experiment.

(b) Find the probability that a white ball is drawn.

(c) Given that a White ball is drawn, find the probability that the box X is used

(4). (a). Explain Followings

(i). the Mean, Mode, Median, Lower quartile, Upper quartile, Standard deviation, coefficient of skewness of the distribution.

(b). Find the mean, median, mode, standard deviation, variance, Lower quartile, upper quartile, and skewness of the following frequency distribution table.

Class	frequency
50-59	2
60-69	2
70-79	5
80-89	9
90-99	14
100-109	19
110-119	15
120-129	10
130-139	3
140-149	1

(5). the logarithmic distribution with parameter $f(x)=\frac{k\theta^x}{x}, x=1,2,3......$

(a). find the constant K as function of θ

(b). show that the mean of this distribution is $\frac{K\theta}{1-\theta}$

(c). by finding $E[x(x-1)]$ or otherwise, show that $E(x^2)=\frac{K\theta}{(1-\theta)^2}$

(6) A particular company produces electrical components utilizing three non-overlapping work shifts. It is observed that 50%, 30% and 20% of the components are produced during shift 1, 2 and 3 respectively. Furthermore 6%, 10% and 8% components produced in shift 1, 2 and 3 respectively are defective. Determine

(a) What percentage of all components is defective?

(b) Given that a defective component is found, what is the probability that is was produced during shift 3?

(c) If one shift has to be closed down which one should it be? What effect would it have on the overall defects to good prices ratio?

Please send your assignment **on or before 05/01/2009** to the following address. **Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.**

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MPZ3230 – Engineering Mathematics I
Assignment No. 04 -2008
Model Answer

(01) (a) $\int_0^1 \frac{1}{1+x}dx \quad h=\frac{1}{4}$

	x	$f(x)=\frac{1}{1+x}$	
x_0	0	$1/(1+0)=1$	f_0
x_1	0.25	$1/(1+0.25)=0.8$	f_1
x_2	0.5	$1/(1+0.5)=0.6667$	f_2
x_3	0.75	$1/(1+0.75)=0.5714$	f_3
x_4	1	$1/(1+1)=0.5$	f_4

Applying Trapezoidal Rule;

$$\int_a^b f(x)\,dx \approx \frac{h}{2}[f_0+2(f_1+f_2+........)+f_n]$$

$$\int_0^1 \frac{1}{1+x}dx \approx \frac{0.25}{2}[1+2(0.8+0.6667+0.5714)+0.5]$$

$$\approx 0.125[1+2(2.0381)+0.5]$$

$$\approx 0.125[1+4.0762+0.5]$$

$$\approx 0.125[5.5762]$$

$$\approx 0.697025$$

(b) $\int_0^5 \frac{1}{4x+5}dx \quad h=0.5$

	x	$f(x)=\frac{1}{4x+5}$	
x_0	0	$1/(4(0)+5)=0.2$	f_0
x_1	0.5	$1/(4(0.5)+5)=0.14286$	f_1
x_2	1.0	$1/(4(1.0)+5)=0.11111$	f_2
x_3	1.5	$1/(4(1.5)+5)=0.09091$	f_3
x_4	2.0	$1/(4(2.0)+5)=0.07692$	f_4
x_5	2.5	$1/(4(2.5)+5)=0.06667$	f_5
x_6	3.0	$1/(4(3.0)+5)=0.05882$	f_6
x_7	3.5	$1/(4(3.5)+5)=0.05263$	f_7
x_8	4.0	$1/(4(4.0)+5)=0.04762$	f_8
x_9	4.5	$1/(4(4.5)+5)=0.04348$	f_9
x_{10}	5.0	$1/(4(5.0)+5)=0.04$	f_{10}

Applying Simpson $\frac{1}{3}$ Rule;

$$\int_a^b f(x)\,dx \approx \frac{h}{3}[f_0+4(f_1+f_3+........+f_{n-1})+2(f_2+f_4+........+f_{n-2})+f_n]$$

$$\int_0^5 \frac{1}{4x+5}dx \approx \frac{0.5}{3}\begin{bmatrix}0.2+4(0.14286+0.09091+0.06667+0.05263+0.04348)+\\ 2(0.11111+0.07692+0.05882+0.04762)+0.04\end{bmatrix}$$

$$\approx \frac{0.5}{3}[0.2+4(0.39655)+2(0.29447)+0.04]$$

$$\approx \frac{0.5}{3}[0.2+1.5862+0.58894+0.04]$$

$$\approx \frac{0.5}{3}[2.41514] \approx 0.40252$$

Also $\int_0^5 \frac{1}{4x+5}dx = \frac{1}{4}\int_0^5 \frac{4}{4x+5}dx$

$$=\frac{1}{4}[\ln|4x+5|]_0^5$$

$$=\frac{1}{4}[\ln 25-\ln 5]$$

$$=\frac{1}{4}\ln\left(\frac{25}{5}\right)$$

$$=\frac{1}{4}\ln 5$$

$$\therefore \frac{1}{4}\ln 5 \approx 0.40252$$

$$\ln 5 \approx 0.40252\times 4$$

$$\ln 5 \approx 1.61008$$

$$\therefore \log_e 5 \approx 1.6101$$

(02)

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
45	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

$x = 52; \quad x_0 = 45; \quad h = 5$

$$s = \frac{x - x_0}{h}$$

$$s = \frac{52-45}{5} = \frac{7}{5} = 1.4$$

From the Newton's forward formula;

$$y_s = y_0 + [\Delta y_0]\binom{s}{1} + [\Delta^2 y_0]\binom{s}{2} + \dots\dots\dots\dots + [\Delta^n y_0]\binom{s}{n}$$

$$y_s = y_0 + [\Delta y_0]\binom{s}{1} + [\Delta^2 y_0]\binom{s}{2} + [\Delta^3 y_0]\binom{s}{3} \text{\_\_\_\_\_\_\_\_} (1)$$

$$\binom{s}{1} = \frac{s!}{1!(s-1)!} = \frac{s(s-1)!}{(s-1)!} = s = 1.4$$

$$\binom{s}{2} = \frac{s!}{2!(s-2)!} = \frac{s(s-1)(s-2)!}{2(s-2)!} = \frac{1.4(1.4-1)}{2} = \frac{1.4\times 0.4}{2} = 0.28$$

$$\binom{s}{3} = \frac{s!}{3!(s-3)!} = \frac{s(s-1)(s-2)(s-3)!}{3\times 2(s-3)!} = \frac{1.4(1.4-1)(1.4-2)}{6} = \frac{1.4\times 0.4(-0.6)}{6} = -0.056$$

From (1)

$$y_{1.4} = 0.7071 + 0.0589\times 1.4 + (-0.0057)(0.28) + (-0.0007)(-0.056)$$

$$y_{1.4} = 0.7071 + 0.08246 - 0.001596 + 0.0000392 = 0.7880032$$

$$\therefore \sin 52^0 = 0.7880$$

(ii) Using Lagrange's method;

$$f(x) = \frac{(x-x_1)(x-x_2)\dots\dots\dots\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots\dots\dots\dots(x_0-x_n)} f(x_0) + \frac{(x-x_0)(x-x_2)\dots\dots\dots\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots\dots\dots\dots(x_1-x_n)} f(x_1)$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)\dots\dots\dots\dots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\dots\dots\dots\dots(x_2-x_n)} f(x_2) + \dots\dots\dots\dots\dots\dots\dots\dots$$
$$+ \frac{(x-x_0)(x-x_1)\dots\dots\dots\dots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\dots\dots\dots\dots(x_n-x_{n-1})} f(x_n)$$

$$f(0.7) = \frac{(0.7-0.2)(0.7-0.3)(0.7-0.4)(0.7-0.5)(0.7-0.6)}{(0.1-0.2)(0.1-0.3)(0.1-0.4)(0.1-0.5)(0.1-0.6)}(2.68)$$
$$+ \frac{(0.7-0.1)(0.7-0.3)(0.7-0.4)(0.7-0.5)(0.7-0.6)}{(0.2-0.1)(0.2-0.3)(0.2-0.4)(0.2-0.5)(0.2-0.6)}(3.04)$$
$$+ \frac{(0.7-0.1)(0.7-0.2)(0.7-0.4)(0.7-0.5)(0.7-0.6)}{(0.3-0.1)(0.3-0.2)(0.3-0.4)(0.3-0.5)(0.3-0.6)}(3.38)$$
$$+ \frac{(0.7-0.1)(0.7-0.2)(0.7-0.3)(0.7-0.5)(0.7-0.6)}{(0.4-0.1)(0.4-0.2)(0.4-0.3)(0.4-0.5)(0.4-0.6)}(3.68)$$
$$+ \frac{(0.7-0.1)(0.7-0.2)(0.7-0.3)(0.7-0.4)(0.7-0.6)}{(0.5-0.1)(0.5-0.2)(0.5-0.3)(0.5-0.4)(0.5-0.6)}(3.96)$$
$$+ \frac{(0.7-0.1)(0.7-0.2)(0.7-0.3)(0.7-0.4)(0.7-0.5)}{(0.6-0.1)(0.6-0.2)(0.6-0.3)(0.6-0.4)(0.6-0.5)}(4.21)$$

$$f(0.7)=\frac{(0.5)(0.4)(0.3)(0.2)(0.1)}{(-0.1)(-0.2)(-0.3)(-0.4)(-0.5)}(2.68)+\frac{(0.6)(0.4)(0.3)(0.2)(0.1)}{(0.1)(-0.1)(-0.2)(-0.3)(-0.4)}(3.04)$$
$$+\frac{(0.6)(0.5)(0.3)(0.2)(0.1)}{(0.2)(0.1)(-0.1)(-0.2)(-0.3)}(3.38)+\frac{(0.6)(0.5)(0.4)(0.2)(0.1)}{(0.3)(0.2)(0.1)(-0.1)(-0.2)}(3.68)$$
$$+\frac{(0.6)(0.5)(0.4)(0.3)(0.1)}{(0.4)(0.3)(0.2)(0.1)(-0.1)}(3.96)+\frac{(0.6)(0.5)(0.4)(0.3)(0.2)}{(0.5)(0.4)(0.3)(0.2)(0.1)}(4.21)$$

$$f(0.7)=-(2.68)+\frac{(0.6)}{0.1}(3.04)+\frac{30}{-2}(3.38)+\frac{120}{6}(3.68)+\frac{30}{-2}(3.96)+6(4.21)$$
$$f(0.7)=-2.68+18.24-50.7+73.6-59.4+25.26$$
$$f(0.7)=4.32$$

(ii) Using Lagrange's inverse interpolation formula;

$$x=\frac{(y-y_1)(y-y_2)\ldots\ldots\ldots\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots\ldots\ldots\ldots(y_0-y_n)}x_0+\frac{(y-y_0)(y-y_2)\ldots\ldots\ldots\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots\ldots\ldots\ldots(y_1-y_n)}x_1$$
$$+\frac{(y-y_0)(y-y_1)(y-y_3)\ldots\ldots\ldots\ldots(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\ldots\ldots\ldots\ldots(y_2-y_n)}x_2+\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+\frac{(y-y_0)(y-y_1)\ldots\ldots\ldots\ldots(y-y_n)}{(y_n-y_0)(y_n-y_1)\ldots\ldots\ldots\ldots(y_n-y_{n+1})}x_n$$

$$x=\frac{(85-87.9)(85-81.3)(85-68.7)}{(94.8-87.9)(94.8-81.3)(94.8-68.7)}(2)+\frac{(85-94.8)(85-81.3)(85-68.7)}{(87.9-94.8)(87.9-81.3)(87.9-68.7)}(5)$$
$$+\frac{(85-94.8)(85-87.9)(85-68.7)}{(81.3-94.8)(81.3-87.9)(81.3-68.7)}(8)+\frac{(85-94.8)(85-87.9)(85-81.3)}{(68.7-94.8)(68.7-87.9)(68.7-81.3)}(14)$$
$$=\frac{(-2.9)(3.7)(16.3)}{(6.9)(13.5)(26.1)}(2)+\frac{(-9.8)(3.7)(16.3)}{(-6.9)(6.6)(19.2)}(5)+\frac{(-9.8)(-2.9)(16.3)}{(-13.5)(-6.6)(12.6)}(8)+\frac{(-9.8)(-2.9)(3.7)}{(-26.1)(-19.2)(-12.6)}(14$$
$$=-.0143878+3.379801+3.301060-0.233153\ =6.30383$$

(ii) Using Lagrange's inverse interpolation formula;

$$x=\frac{(y-y_1)(y-y_2)\ldots\ldots\ldots\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots\ldots\ldots\ldots(y_0-y_n)}x_0+\frac{(y-y_0)(y-y_2)\ldots\ldots\ldots\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots\ldots\ldots\ldots(y_1-y_n)}x_1$$
$$+\frac{(y-y_0)(y-y_1)(y-y_3)\ldots\ldots\ldots\ldots(y-y_n)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)\ldots\ldots\ldots\ldots(y_2-y_n)}x_2+\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+\frac{(y-y_0)(y-y_1)\ldots\ldots\ldots\ldots(y-y_n)}{(y_n-y_0)(y_n-y_1)\ldots\ldots\ldots\ldots(y_n-y_{n+1})}x_n$$

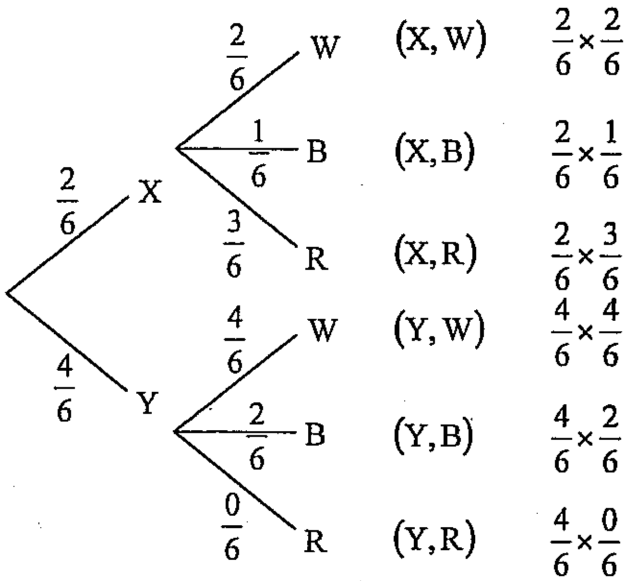
$$x=\frac{(13)(-3)(-18)(30)}{(-17)(-33)(-48)}+\frac{(30)(-3)(-18)(34)}{(17)(-16)(-31)}+\frac{(30)(13)(-18)(38)}{(33)(16)(-15)}+\frac{(30)(13)(-3)(42)}{(48)(31)(15)}$$
$$x=-0.782086+6.532258+33.681818-2.201613\ =37.230377$$

(03) X Y score the box taken the ball

X	Y	score
W = 2	W = 4	$1/6 \rightarrow X$
B = 1	B = 4	$2/3/4/5 \rightarrow Y$
R = 3	R = 0	

When the score is 1 or 6 use box X = X
When the score is 2, 3, 4 or 5 use box Y = Y
When the color of ball is White = W
When the color of ball is Blue = B
When the color of ball is Red = R

(a)



(b) The probability that a White ball is drawn out $P(W)=\frac{2}{6}\times\frac{2}{6}+\frac{4}{6}\times\frac{4}{6}=\frac{4}{36}+\frac{16}{36}=\frac{20}{36}=\frac{5}{9}$

(c) The probability of White ball drawn $P(W)=\frac{5}{9}$

The probability of White ball drawn from X box $P(W/x)=\frac{2}{6}\times\frac{2}{6}=\frac{4}{36}=\frac{1}{9}$

If given that a White ball is drawn, the probability that the X box is used $=\frac{P(W/X)}{P(W)}=\frac{1/9}{5/9}$

$=\frac{1}{9}\times\frac{9}{5}=\frac{1}{5}$

(04) (a)(i)

The Mean $\bar{x}$

The mean or Average of a number of values is defined as,

$\text{mean}(\bar{x})=\frac{\text{sum of the values}}{\text{number of values}}$

This equation can be used for ungrouped data. If the data of a sample of size n is given by $\{x_1, x_2, \ldots\ldots\ldots x_n\}$ the $\bar{x}$ can be expressed as $\bar{x}=\frac{x_1+x_2+\ldots\ldots\ldots+x_n}{n}=\frac{\sum_{i=1}^{n} x}{n}$ we can't use this formula if the data is presented as grouped data. In this case we can use bellow formula.

$\bar{x}=A+c\frac{\sum f_i d_i}{n}$ where $d_i=\frac{m_i-A}{C}$

Where A - class mid point some where in the middle of the table.
c - class width
m_i - class mid point

f_i - frequency

The mode

mode is the most likely value or the value with the highest frequency. For ungrouped data we can find the mode by finding the value with highest frequency from the array. For grouped data we can obtain the mode by the formula.

$$\text{mode} = L + \left[\frac{d_1}{d_1 + d_2}\right] \times w$$

L - True lower class limit of modal class
w - width of modal class
d_1 - frequency of modal class - frequency of previous class
d_2 - frequency of modal class - frequency of next class
modal class = class with highest frequency

The Median

Median is the mid value of data. For ungrouped data, if the sample size n is a odd number thus the median is given by ; median $= \frac{(n+1)^{th}}{2}$ value of the array.

If the sample size n is an even number then the median is given by ;

$$\text{median} = \frac{1}{2}\left[\frac{n^{th}}{2}\text{value} + \text{next value}\right]$$

For grouped data we can expressed following formula. ; median $= L + \left[\frac{n/2 - F_1}{f}\right] \times w$

where
L - True lower class limit of median class
f - Frequency of median class
w - Width of median class
F_1 - Cumulative frequency of class before median

Lower quartile (Q_1)

The interpretation of the median is that 50% (equal number) of the data have values less than the median. So the new median of that 50% values is the lower quartile and for small set of values $Q_1 = \frac{1}{4}(n+1)^{th}$ value and for grouped data $Q_1 = \left(\frac{1}{4}n\right)^{th}$ value. Q_1 can be calculated by using the above median formula by changing the median class to Q_1 class & changing $\frac{n}{2}$ to $\frac{n}{4}$.

Upper quartile (Q_3)

The interpretation of the median is that 50% of the data have values more than the median. So the new median of that 50% values is the upper quartile and for small set of values $Q_3 = \frac{3}{4}(n+1)^{th}$ value and for grouped data $Q_3 = \left(\frac{3}{4}n\right)^{th}$ value. This Q_3 can be calculated by using the above median formula by changing the median class to Q_3 class & changing $\frac{n}{2}$ to $\frac{3n}{4}$.

Standard deviation

Standard deviation is definite as the root mean square deviation (from mean) of the data. That standard deviation is the square root of mean of the square deviation of the data from mean. We expressed standard deviation for grouped data as

$$S = c\sqrt{\frac{\sum_{j=1}^{k}\left(f_j d_j^{\,2}\right)}{n} - \left(\bar{d}\right)^2} \qquad d_j = \frac{m_j - A}{c} \qquad \bar{d} = \frac{\sum_{j=1}^{k} f_j d_j}{n}$$

m_j - class mid point $\quad$ f_j - frequency $\quad$ c - class width
A - class mid point some where in the middle of the table

Coefficient of skewness of the distribution

skewness is the degree of deviation of the frequency curve (polygon) from symmetry skewness can positive or negative. There are two measures of skewness.

(i) Pearson's coefficient of skewness (S_{kp})

$$S_{kp} = \frac{\text{mean} - \text{mode}}{\text{standard deviation}}$$

(ii) Bowley's coefficient of skewness (S_{kq})

$$S_{kq} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

Q_1-Lower quartile
Q_2-The Median
Q_3-Upper quartile

If the frequency curve is symmetrical then

(i) mean = mode = median

(ii) $Q_3 - Q_2 = Q_2 - Q_1$

$$\therefore S_{kp} = \frac{\text{mean} - \text{mode}}{\text{standard deviation}} = 0$$

$$S_{kq} = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_1)} = 0$$

$\therefore S_{kp}$ & S_{kp} are zero for symmetrical distribution.

class	M	f_i	F_i	$d_i - \frac{x_i - A}{c}$	$f_i d_i$	$f_i d_i^2$
50 - 59	54.5	2	2	-5	-10	50
60 - 69	64.5	2	4	-4	-08	32
70 - 79	74.5	5	9	-3	-15	45
80 - 89	84.5	9	18	-2	-18	36
90 - 99	94.5	14	32	-1	-14	14
100 - 109	104.5	19	51	0	0	0
110 - 119	114.5	15	66	1	15	15
120 - 129	124.5	10	76	2	20	40
130 - 139	134.5	3	79	3	09	27
140 - 149	144.5	1	80	4	04	16
		80			48 - 65 = -17	275

$$\text{mean } \bar{x} = A + c\frac{\sum f_i d_i}{n}$$

$$= 104.5 + 10\frac{(-17)}{80} = 104.5 - 2.125$$

$= 102.375$

median class $= 100 - 109$

$$\text{median} = L + \left[\frac{n/2 - F_1}{f}\right] \times w$$

$$= 99.5 + \left[\frac{80/2 - 32}{19}\right] \times 10$$

$$\text{median} = 99.5 + \left[\frac{40 - 32}{19}\right] \times 10$$

$$= 99.5 + \frac{180}{19} = 99.5 + 4.2105$$

$$= 103.7105$$

$$\text{modal class} = 100 - 109$$

$$\text{mode} = L + \left[\frac{d_1}{d_1 + d_2}\right] \times w$$

$$= 99.5 + \left[\frac{5}{5+4}\right] \times 10 = 99.5 + \frac{50}{9}$$

$$= 99.5 + 5.5556 = 105.0556$$

$$\text{Standard deviation } (S) = c\sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2} = 10\sqrt{\frac{275}{80} - \left(\frac{-17}{80}\right)^2}$$

$$= 10\sqrt{\frac{55}{16} - \frac{289}{6400}} = 10\sqrt{3.4375 - 0.04515625} = 10\sqrt{3.39234375} = 18.4183$$

$$\text{Variance}(S^2) = (18.4183)^2 = 339.2338$$

$$\text{lower quartile class} = \frac{1}{4}n^{th} \text{ value class} = 90 - 99$$

$$Q_1 = L + \left[\frac{n/4 - F_1}{f}\right] \times w = 89.5 + \left[\frac{80/4 - 18}{14}\right] \times 10 = 89.5 + \left[\frac{20 - 18}{14}\right] \times 10 = 89.5 + \frac{2}{14} \times 10$$

$$= 89.5 + \frac{10}{7} = 89.5 + 1.4286 = 90.9286$$

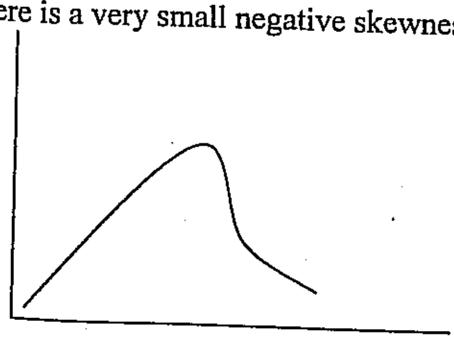
$$\text{upper quartile class} = \frac{3}{4}n^{th} \text{ value class} = 60^{th} \text{ value class} = 110 - 119$$

$$Q_3 = L + \left[\frac{3n/4 - F_1}{f}\right] \times w = 109.5 + \left[\frac{3/4(80) - 51}{15}\right] \times 10 = 109.5 + \left[\frac{60 - 51}{15}\right] \times 10$$

$$= 109.5 + \frac{9}{15} \times 10 = 109.5 + 6 = 115.5$$

$$\text{Coefficient of skewness } (S_{kp}) = \frac{\text{mean} - \text{mode}}{\text{standard deviation}} = \frac{102.375 - 105.0556}{18.4183} = -0.1455$$

There is a very small negative skewness. That is skewness to left.



(05) $f(x)=\frac{k\theta^x}{x}; \quad x=1,2,3,\ldots\ldots\ldots\ldots$

Above logarithmic distribution function f(x) of a discrete r.v. x (that is satisfy two conditions)

(i) $0\le f(x)\le 1$ for any x &

(ii) $\sum_{\text{all } x} f(x)=1$

Using the second condition;

$$\sum_{x=1}^{\infty} f(x)=1$$

$$\sum_{x=1}^{\infty}\frac{k\theta^x}{x}=1$$

$$k\sum_{x=1}^{\infty}\frac{\theta^x}{x}=1$$

$$k\left[\theta+\frac{\theta^2}{2}+\frac{\theta^3}{3}+\ldots\ldots\ldots\ldots\right]=1$$

Using Taylor's series method;

$$\theta+\frac{\theta^2}{2}+\frac{\theta^3}{3}+\ldots\ldots\ldots\ldots=-\ln(1-\theta);\ -1<\theta<1;\ k=\frac{-1}{\ln(1-\theta)}$$

(b) The mean of this distribution is defined as,

$E(x)=\sum_{\text{all } x} x\,f(x)$ because it is a function of a discrete r.v. x

$$E(x)=\sum_{x=1}^{\infty} x\frac{k\theta^x}{x}$$

$$E(x)=\sum_{x=1}^{\infty} k\theta^x = k\sum_{x=1}^{\infty}\theta^x$$

$$E(x)=k\left[\theta+\theta^2+\theta^3+\ldots\ldots\ldots\ldots\right] \quad \text{(1)}$$

$$S_n=\theta+\theta^2+\theta^3+\ldots\ldots\ldots\ldots+\theta^{n-1}+\theta^n$$

$$\theta S_n=\theta^2+\theta^3+\theta^4+\ldots\ldots\ldots\ldots+\theta^{n-1}+\theta^n+\theta^{n+1}$$

$$(1-\theta)S_n=\theta-\theta^{n+1}$$

$$S_n=\frac{\theta(1-\theta^n)}{(1-\theta)}$$

$$\lim_{n\to\infty} S_n=\lim_{n\to\infty}\frac{\theta(1-\theta^n)}{(1-\theta)}$$

But $-1<\theta\le 1$ $\therefore \theta$ is a very small value.

$$\therefore \theta^\infty \to 0$$

$$\lim_{n\to\infty} S_n=\frac{\theta}{1-\theta}$$

From (1) $E(x)=\frac{k\theta}{(1-\theta)}$

(c) For a discrete r.w. x;

$$E(x)=\sum_{\text{all } x} x\,f(x)$$

$$E[x(x-1)]=\sum_{x=1}^{\infty} x(x-1)\frac{k\theta^x}{x}$$

$$E[x^2-x]=k\sum_{x=1}^{\infty}(x-1)\theta^x$$

$$E(x^2)-E(x)=k\left[\sum_{x=1}^{\infty}x\theta^x-\sum_{x=1}^{\infty}\theta^x\right]$$

$$E(x^2)-E(x)=k\sum_{x=1}^{\infty}x\theta^x-k\sum_{x=1}^{\infty}\theta^x$$

$$E(x^2)-E(x)=k\sum_{x=1}^{\infty}x\theta^x-E(x)$$

$$E(x^2)=k\left[\theta+2\theta^2+3\theta^3+\ldots\ldots\ldots\ldots\right]$$

$$S_n' = \theta + 2\theta^2 + 3\theta^3 + 4\theta^4 + \dots\dots + (n-1)\theta^{n-1} + n\theta^n$$

$$\theta S_n' = \theta^2 + 2\theta^3 + 3\theta^4 + \dots\dots + (n-2)\theta^{n-1} + (n-1)\theta^n + n\theta^{n+1}$$

$$(1-\theta)S_n' = \underbrace{\theta + \theta^2 + \theta^3 + \theta^4 + \dots\dots + \theta^{n-1} + \theta^n - n\theta^{n+1}}_{S_n}$$

$$(1-\theta)S_n' = \frac{\theta(1-\theta^n)}{1-\theta} - n\theta^{n+1}$$

$$S_n' = \frac{\theta(1-\theta^n) - (1-\theta)n\theta^{n+1}}{(1-\theta)^2} = \frac{\theta - \theta^n[\theta + n\theta(1-\theta)]}{(1-\theta)^2}$$

$$\lim_{n\to\infty} S_n' = \lim_{n\to\infty} \frac{\theta - \theta^n[\theta + n\theta(1-\theta)]}{(1-\theta)^2}$$

But $-1 < \theta \le 1$ $\quad \therefore \theta$ is a very small value.

$\therefore \theta^\infty \to 0$

$$\lim_{n\to\infty} S_n' = \frac{\theta}{(1-\theta)^2}$$

From (A)

$$E(x^2) = \frac{k\theta}{(1-\theta)^2}$$

(06) Producing a component during shift $1 \to A$
Producing a component during shift $2 \to B$
Producing a component during shift $3 \to C$
When the produced component is defective $\to D$
When the produced component is non defective $\to E$

$\therefore P(A) = \frac{50}{100} = 0.5 \qquad P(D/A) = \frac{6}{100} = 0.06$

$P(B) = \frac{30}{100} = 0.3 \qquad P(D/B) = \frac{10}{100} = 0.1$

$P(C) = \frac{20}{100} = 0.2 \qquad P(D/C) = \frac{8}{100} = 0.08$

(a) $P(D) = P(D/A).P(A) + P(D/B).P(B) + P(D/C).P(C)$

$P(D) = 0.06 \times 0.5 + 0.1 \times 0.3 + 0.08 \times 0.2$

$P(D) = 0.03 + 0.03 + 0.016$

$P(D) = 0.076$

The percentage of all components is defective $= 0.076 \times 100 = 7.6\ \%$

(b) Using Baye's Theorem,

$$P(C/D) = \frac{P(D/C).P(C)}{P(D/A).P(A) + P(D/B).P(B) + P(D/C).P(C)}$$

$$= \frac{0.08 \times 0.2}{0.06 \times 0.5 + 0.1 \times 0.3 + 0.08 \times 0.2} = \frac{0.016}{0.076} = 0.2105$$

Given that a defective component is found, the probability that it was produced during shift 3 is $= 0.2105$

(c) $$P(A/D) = \frac{P(D/A).P(A)}{P(D/A).P(A) + P(D/B).P(B) + P(D/C).P(C)}$$

$$= \frac{0.06 \times 0.5}{0.06 \times 0.5 + 0.1 \times 0.3 + 0.08 \times 0.2} = \frac{0.03}{0.076} = 0.3947$$

$$P(B/D) = \frac{P(D/B).P(B)}{P(D/A).P(A) + P(D/B).P(B) + P(D/C).P(C)}$$

$$= \frac{0.1 \times 0.3}{0.06 \times 0.5 + 0.1 \times 0.3 + 0.08 \times 0.2} = \frac{0.03}{0.076} = 0.3947$$

If we arbitrary take a defective component, the probability that it was produced during shift 1 & shift 2 is same and higher than the corresponding value of shift 3. But shift 1 produce non defective components more than the shift 2 produce.
So the shift which has to be closed down should be shift 2.
$P(E/A) = 0.94$ $\quad P(E/B) = 0.9$ $\quad P(E/C) = 0.92$

Before close the shift
$$P(E) = P(E/A).P(A) + P(E/B).P(B) + P(E/C).P(C)$$
$$= 0.94 \times 0.5 + 0.9 \times 0.3 + 0.92 \times 0.2 = 0.47 + 0.27 + 0.184 = 0.924$$

After close the shift
$$P(E) = P(E/A).P(A) + P(E/C).P(C) = 0.94 \times 0.5 + 0.92 \times 0.2 = 0.654$$

$$\text{Probability of effect} = \frac{0.654}{0.924} = 0.708$$