MPZ 3230 – Engineering Mathematics I

Assignment No. 01 –Academic Year - 2008

Answer all questions

(1). Write down dot product and scalar products of two vectors.

Write an expression for the angle between two 3D, vectors $\underline{a}$ and $\underline{b}$.

A, B, C and D are four points in space. They have position vectors $\underline{a},\underline{b},\underline{c}$ and $\underline{d}$ respectively,

Where, $\underline{a}=(2,4,-1)$ $\quad \underline{b}=(4,1,0)$

$\underline{c}=(1,2,2)$ $\quad \underline{d}=(3,3,3)$

(a). Find vectors $\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{BC}\ and\ \overrightarrow{CD}$

(b). show that ABC is an equilateral triangle.

(c). Find $\overrightarrow{AB}\times\overrightarrow{BC}$ and hence deduce the area of triangle ABC

(d). Find the value of K where, $K=\left(\overrightarrow{AB}\times\overrightarrow{BC}\right)\left(\overrightarrow{CA}\right)$ using the value of k, what can you say about points A,B, and C?

(e). the shortest distance 'ℓ' between two lines AB and CD is given by, $\ell=\frac{\underline{n}}{n}.\left(\overrightarrow{AC}\right)$

Where $\underline{n}=\left(\overrightarrow{AB}\times\overrightarrow{CD}\right)$ find this shortest distance ℓ

(2).

(i). Find the unit vector perpendicular to each of the vectors,

$\underline{a}=2\underline{i}-\underline{j}+\underline{k}$ $\quad \underline{b}=3\underline{i}+4\underline{j}-\underline{k}$

Calculate the angle between them.

(ii). Show that the following vectors are coplanar,

(i). $\underline{a}=2\underline{i}-\underline{j}+\underline{k},\underline{b}=\underline{i}+2\underline{j}-3\underline{k}\ and\ \underline{c}=3\underline{i}-4\underline{j}+5\underline{k}$

(ii). $\underline{a}=\underline{j}-2\underline{k},\underline{b}=\underline{i}-\underline{j}+\underline{k},\underline{c}=-2\underline{i}+3\underline{j}-4\underline{k}$

(3).

(i) (a) Compute the following scalar triple products.

(i). $(\underline{i}-2\underline{j}+3\underline{k})\times(2\underline{i}+\underline{j}-\underline{k})(\underline{j}+\underline{k})$

(ii). $(2\underline{i}-3\underline{j}+k)(\underline{i}-\underline{j}+2\underline{k})\wedge(2\underline{i}+\underline{j}-\underline{k})$

(b). Compute the following vector products.

(i). $[(\underline{i}-\underline{j}+\underline{k})\wedge(2\underline{i}-3\underline{j}-k)]\wedge[(-3\underline{i}+j+\underline{k})\wedge(2\underline{j}+\underline{k})]$

(ii). $[(3\underline{i}-2\underline{j}-2\underline{k})\wedge(\underline{i}-k)]\wedge[(\underline{i}+j+\underline{k})\wedge(\underline{i}-2\underline{j}+3\underline{k})]$

(ii). Prove,

$$\frac{d(\underline{a}\times\underline{b}\times\underline{c})}{dt}=\frac{d\underline{a}}{dt}\times\underline{b}\times\underline{c}+\underline{a}\times\frac{d\underline{b}}{dt}\times\underline{c}+\underline{a}\times\underline{b}\times\frac{d\underline{c}}{dt}.$$

(4).

(i). Show that,

$r=e^{-t}(c_1\cos 2t+c_2\sin 2t)$, where C_1 and C_2 are constant vectors, is a solution of the differential equation.

$$\frac{d^2r}{dt^2}+2\frac{dr}{dt}+5r=0$$

(ii). Solve the following differential equations. Using suitable method,

(i). $(xy+x)dy-(xy+y)dx=0 \quad sol: x=cye^{y-x}$

(ii). $(x+1)\frac{dy}{dx}+1=2e^{-y} \quad sol: c(x+1)(2-e^{y})=1$

(iii). $(e^{y}+2)\sin x\,dx-e^{y}\cos x\,dy=0 \quad sol: (e^{y}+2)\cos x=C$

(iv). $x(y-x)\frac{dy}{dx}=y(y+x) \quad sol: y/x-\log xty=a$

(5). Solve the following differential equation using exact method,

(i). $(x+y-10)dx+(x-y-2)dy=0$

(ii). $(e^{y}+2)\sin x dx-e^{y}\cos x dy=0$

(iii). Find the value of K, for which the differential equation,
$(xy^2+kx^2y)dx+(x+y)x^2dy=0$ is exact, solve the equation for this value of K,

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Please send your assignment **on or before the due date** to the following address. **Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.**

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MPZ 3230
Model answer No. 01 – Academic Year 2008

(1). the scalar, or dot product of two vectors a and b is defined to be $|\underline{a}|\ |\underline{b}|\ \cos\theta$ (a scalar) where θ is the angle between a and b.

Symbolically $\underline{a}.\underline{b} = |\underline{a}|\ |\underline{b}|\ \cos\theta$

Due to a dot between a and b. this product is also called dot product,

Angle between $\underline{a}$ and $\underline{b}$ is, $\cos\theta = \frac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$

$\underline{a} = (2,4,-1), \quad \underline{b} = (4,1,0)$

$\underline{c} = (1,2,2), \quad \underline{d} = (3,3,3)$

(a). $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= (-2,-4,1) + (4,1,0)$

$= (2,-3,1)$

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = (-2,-4,1) + (1,2,2)$

$= (-1,-2,3)$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC} = (-4,-1,0) + (1,2,2)$

$= (-3,1,2)$

$\overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD} = (-1,-2,-2) + (3,3,3)$

$= (2,1,1)$

(b).If ABC are equivalent triangle.

$|AB| = |BC| = |CA|$

$\overrightarrow{AB} = (2,-3,1), |AB| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$

$\overrightarrow{BC} = (-3,1,2), |BC| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}$

$\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} = (-1,-2,-2) + (2,4,-1) = (1,2,-3), |\overrightarrow{CA}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

$\therefore |AB| = |BC| = |CA|$

$\therefore$ ABC are equilateral triangle.

(c). $\overrightarrow{AB} \times \overrightarrow{BC}$

$\overrightarrow{AB} = (2,-3,1) \qquad \overrightarrow{BC} = (-3,1,2)$

$$\overrightarrow{AB}\times\overrightarrow{BC}=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -3 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$

$$=\underline{i}(-6-1)-\underline{j}(4+3)+\underline{k}(2-9)$$

$$=-7\underline{i}-7\underline{j}-7\underline{k}$$

$$\overrightarrow{AB}\times\overrightarrow{BC}=-7\underline{i}-7\underline{j}-7\underline{k}$$

Area of triangle $=\frac{1}{2}\left|\left(\overrightarrow{AB}\times\overrightarrow{BC}\right)\right|$

$$=\frac{1}{2}\left|(-7,-7,-7)\right|$$

$$=\frac{1}{2}\sqrt{7^2+7^2+7^2}$$

$$=\frac{\sqrt{49+49+49}}{2}$$

=6.062 units

(d). $k=\left(\overrightarrow{AB}\times\overrightarrow{BC}\right).\left(\overrightarrow{CA}\right)$

$\left(\overrightarrow{AB}\times\overrightarrow{BC}\right)=(-7,-7,-7)$ [by using part (c)]

$\overrightarrow{CA}=(1,2,-3)$ [by using part (b)]

$$\left(\overrightarrow{AB}\times\overrightarrow{BC}\right)\left(\overrightarrow{CA}\right)=-(-7,-7,-7)\times(1,2,-3)$$

$$=-7-14+21$$

$$=0$$

$$\therefore k=0$$

We can say A, B, C points are coplanar.

(e). $\underline{\ell}=\frac{\underline{n}}{n}.\left(\overrightarrow{AC}\right)$

Given $\underline{n}=\left(\overrightarrow{AB}\times\overrightarrow{CD}\right)$ [by using part (a)]

$$\overrightarrow{AB}\times\overrightarrow{CD}=\begin{vmatrix} \underline{i} & \underline{j} & \underline{F} \\ 2 & -3 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$\overrightarrow{AB}=(2,-3,1)$
$\overrightarrow{CD}=(2,1,1)$
$\overrightarrow{AC}=(-1,-2,3)$

$$=\underline{i}(-3-1)-\underline{j}(2-2)+\underline{k}(2+6)$$

$$=(-4,0,8)$$

$$\underline{n}=(-4,0,8)$$

$$|n|=\sqrt{16+0+34}=\sqrt{80}$$

$$\underline{\ell} = \frac{\underline{n}}{|\underline{n}|}.(\overrightarrow{AC})$$

$$= \frac{1}{\sqrt{80}}(-4,0,8).(-1,-2,3)$$

$$= \frac{1}{\sqrt{80}}(4+0+24)$$

$$= \frac{28}{\sqrt{80}} = \frac{28}{\sqrt{4\times4\times5}}$$

$$\ell = 7/\sqrt{5}$$

=3.13 units

(2). Unit vector (perpendicular broth $\underline{a}$ and $\underline{b}$) $= \frac{\underline{a}\times\underline{b}}{|a\times b|}$

$$\underline{a} = (1,-1,1)$$

$$\underline{b} = (3,4,-1)$$

$$\underline{a}\times\underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$$

$$= \underline{i}(1-4) - \underline{j}(-2-3) + \underline{k}(8+3)$$

$$= (-3\ ,5\ \ 11)$$

$$\underline{a}\times\underline{b} = (-3,5,11)$$

$$|a\times b| = \sqrt{3^2+5^2+11^2} = \sqrt{9+25+121} = \sqrt{155}$$

Unit vector $= \underline{n}$

$$\underline{n} = \frac{\underline{a}\times\underline{b}}{|\underline{a}\times\underline{b}|}$$

$$= \frac{(-3,5,11)}{\sqrt{155}}$$

$$\underline{n} = \frac{-3}{\sqrt{155}}\underline{i} + \frac{5}{\sqrt{155}}\underline{j} + \frac{11}{\sqrt{155}}\underline{k}$$

Calculate angle between $\underline{a}$ and $\underline{b}$

$$\underline{a}.\underline{b} = |a|\ |b|\ \cos\theta$$

Angle is $\cos\theta = \frac{a.b}{|a||b|}$

$$\theta = \cos^{-1}\left(\frac{\underline{a}.\underline{b}}{|\underline{a}|\ |\underline{b}|}\right)$$

$$\underline{a} = (2,-1,1) \quad \underline{b} = (3,4,-1)$$

$$\underline{a}.\underline{b} = (2,-1,1).(3,4,-1) \quad = 6-4-1 = 1$$

$$|a| = \sqrt{2^2+1^2+1^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|b| = \sqrt{3^2+4^2+1^2} = \sqrt{9+16+1} = \sqrt{26}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{26}}\right)$$

$$\theta = 85.403^o$$

(ii).

(a) Find which are coplanar.

$\underline{a} = (2,-1,1) \quad ,\underline{b} = (1,2,-3)$

$\underline{c} = (3,-4,5) \quad ,\underline{d} = (2,1,-3)$

$$\overrightarrow{AB} = \underline{b} - \underline{a} = (1, 2, -3) - (2, -1, 1) = (-1, 3, -4)$$

$$\overrightarrow{AC} = \underline{c} - \underline{a} = (3, -4, 5) - (2, -1, 1) = (1, -3, 4)$$

$$\overrightarrow{AD} = \underline{d} - \underline{a} = (2, 1, -3) - (2, -1, 1) = (0, 2, -4)$$

$$\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\overrightarrow{AD} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -1 & 3 & -4 \\ 1 & -3 & 4 \end{vmatrix} .(0, 2, -4)$$

$$= \left[\underline{i}(12-12) - \underline{j}(-4+4) + \underline{k}(3-3)\right](0, 2, -4)$$

$$= (-0,0,0) \times (0, 2, -4)$$

$$= 0$$

$\therefore$ a, b, c, d are coplanar. $\therefore \underline{a}, \underline{b}, \underline{c}, \underline{d}$ Points are same plan

(b) $\underline{a} = (0,1,-2) \quad \underline{b} = (1, -1, 1)$

$\underline{c} = (-2, 3, -4) \quad \underline{d} = (2, 1, -1)$

$$\overrightarrow{AB} = \underline{b} - \underline{a} = (1, -1, 1) - (0, 1, -2) = (1, -2, 3)$$

$$\overrightarrow{AC} = \underline{c} - \underline{a} = (-2, 3, -4) - (0, 1, -2) = (-2, 2, -2)$$

$$\overrightarrow{AD} = \underline{d} - \underline{a} = (2,1,-1) - (0, 1, -2) = (2, 0,1)$$

$$\left(\overrightarrow{AB} \times \overrightarrow{AC}\right)\overrightarrow{AD} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -2 & 3 \\ -2 & 2 & -2 \end{vmatrix} .(2, 0, 1)$$

$$= [\underline{i}(4-6)-\underline{j}(-1+6)+\underline{k}(2-4)].(2,\ 0,1)$$
$$= (-2,\ 5,\ -2).(2,\ 0,\ 1)$$
$$= -4+0-2 = -6(\neq 0)$$

$\therefore$ a, b, c, d points are not coplanar.

$\therefore$ Can check three points. Those are same plan.

(a) $\underline{a}=(2,-1,1)$ $\underline{b}=(1,2,-3)$

$\underline{c}=(3,-4,5)$ $\underline{d}=(2,\ 1,-3)$

Check whether three points,

$$\text{(i). } (a\times b).c = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 1 \\ 1 & 2 & -3 \end{vmatrix}.(3,-4,\ 5)$$
$$= [\underline{i}(3-2)-\underline{j}(-6-1)+\underline{k}(4+1)](3,\ -4,\ 5)$$
$$= (1,\ 7,\ 5).(3,\ -4,5)$$
$$= 3-28+25$$
$$= 0$$

$\underline{a},\underline{b},\underline{c}$ Points are co-planer. $\therefore$ $\underline{a},\underline{b}$ and $\underline{c}$ Points are same plan.

(ii). $(\underline{a}\times \underline{b}).\underline{d} = (1,\ 7,\ 5).(2,\ 1,\ -3)$ $[\underline{a}\times\underline{b}=(1,7,5)]$(by using part (i))

$$= 2+7-15$$
$$= -6(\neq 0)$$

a, b, d are not coplanar $\therefore$ a, b, d are not same plan

$$\text{(iii). } (\underline{a}\times \underline{c}).\underline{d} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & -1 & 1 \\ 3 & -4 & 5 \end{vmatrix}.(2,\ 1,\ -3)$$
$$= [\underline{i}(-5+4)-\underline{j}(10-3)+\underline{k}(-8+3)](2,+1,-3)$$
$$= (-1,\ -7,\ -5).(2,\ 1,\ -3)$$
$$= -2-7+15$$
$$= 6(\neq 0)$$

a ,c ,d are not coplanar $\therefore$ a , c ,d , are not same plan.

$$\text{(iv). } (\underline{b}\times \underline{c}).\underline{d} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 2 & -3 \\ 3 & -4 & 5 \end{vmatrix}.(2,1,-3)$$
$$= [\underline{i}(10-12)-\underline{j}(5+9)+\underline{k}(-4-6)](2,1,-3)$$
$$= (-2,-14,-10).(2,1,-3)$$
$$= -4-14+30$$
$$= 12,(\neq 0)$$

$\underline{b}$, $\underline{c}$,$\underline{d}$ are not coplanar $\therefore$ $\underline{b}$, $\underline{c}$, $\underline{d}$ points also not same plan.

(b) $\underline{a} = (0, 1, -2)$ $\quad \underline{b} = (1, -1, 1)$

$\underline{c} = (-2, 3 -4)$ $\quad \underline{d} = (2, 1, -1)$

(i) $(\underline{a} \times \underline{b}).\underline{c} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & -2 \\ 1 & -1 & 1 \end{vmatrix}.(-2, 3, -4)$

$= [\underline{i}(1-2) - \underline{j}(0+2) + \underline{k}(0-1)].(-2, 3, -4)$

$= (-1, -2, -1).(-2, 3, -4)$

$= 2 - 6 + 4$

$= 0$

$\therefore$ $\underline{a}$,$\underline{b}$,$\underline{c}$, are coplanar. $\therefore$ $\underline{a}$,$\underline{b}$,$\underline{c}$, points are same plan.

(ii). $(\underline{a} \times \underline{b}).\underline{d} = (-1,-2,-1).(2,1,-1)$ $\quad$ [by using part (i) $\underline{a} \times \underline{b} = (-1,-2,-1)$]

$= -2 - 2 + 1$

$= -3 (\neq 0)$

$\underline{a}$, $\underline{b}$, $\underline{d}$, are not coplanar $\therefore$ $\underline{a}$, $\underline{b}$, $\underline{d}$ are not same plan.

(iii). $(\underline{a} \times \underline{c}).\underline{d} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & -2 \\ -2 & 3 & -4 \end{vmatrix}.(2,1,-1)$

$= [\underline{i}(-4+6) - \underline{j}(0-4) + \underline{k}(0+2)].(2,1,-1)$

$= (2,4,2).(2,1,-1)$

$= 4 + 4 - 2$

$= 6 (\neq 0)$

$\therefore$ $\underline{a}$, $\underline{b}$, $\underline{d}$ are not coplanar $\therefore$ $\underline{a}$, $\underline{c}$, $\underline{d}$ are not same plan.

(iv). $(\underline{b} \times \underline{c}).\underline{d} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 1 \\ -2 & 3 & -4 \end{vmatrix}.(2,1,-1)$

$= [\underline{i}(4-3) - \underline{j}(-4+2) + \underline{k}(3-2)].(2,1,-1)$

$= (1,2,1).(2,1,-1)$

$= 2 + 2 - 1 \quad = 3 (\neq 0)$

$\underline{b}$, $\underline{c}$, $\underline{d}$ are not coplanar $\therefore$ $\underline{b}$, $\underline{c}$, $\underline{d}$ are not same plan.

(3). (i). $|(\underline{i} - \underline{j} + \underline{k}) \wedge (2\underline{i} - 3\underline{j} - \underline{k})| \wedge |(-3\underline{i} + \underline{j} + \underline{k}) \wedge (2\underline{j} + \underline{k})|$

$$=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -1 & 1 \\ 2 & -3 & -1 \end{vmatrix} \wedge \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ -3 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

$$=\left[\underline{i}(1+3)-\underline{j}(-1-2)+\underline{k}(-3+2)\right]\wedge\left[\underline{i}(1-2)-\underline{j}(-3-0)+\underline{k}(-6-0)\right]$$

$$=(4, 3, -1)\wedge(-1, 3, -6)$$

$$=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 4 & 3 & -1 \\ -1 & 3 & -6 \end{vmatrix}$$

$$=i(-18+3)-\underline{j}(-24-1)+\underline{k}(12+3)$$

$$=(-15, 25, 15)$$

(ii). $\left[(3\underline{i}-2\underline{j}-2\underline{k})\wedge(\underline{i}-\underline{k})\right]\wedge\left[(\underline{i}+\underline{j}+\underline{k})\wedge(\underline{i}-2\underline{j}+3\underline{k})\right]$

$$=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 3 & -2 & -2 \\ 1 & 0 & -1 \end{vmatrix} \wedge \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$=\left[\underline{i}(2-0)-\underline{j}(-3+2)+\underline{k}(0+2)\right]\wedge\left[\underline{i}(3+2)-\underline{j}(3-1)+\underline{k}(-2-1)\right]$$

$$=(2, 1, 2)\wedge(5, -2, -3)$$

$$=\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & 2 \\ 5 & -2 & -3 \end{vmatrix}$$

$$=\underline{i}(-3+4)-\underline{j}(-6-10)+\underline{k}(-4-5)$$

$$=(1, +16, -9)$$

(ii). $\dfrac{d(\underline{a}\times\underline{b}\times\underline{c})}{dt}=\dfrac{d\underline{a}}{dt}\times\underline{b}\times\underline{c}+\underline{a}\times\dfrac{d\underline{b}}{dt}\times\underline{c}+\underline{a}\times\underline{b}\dfrac{d\underline{c}}{dt}$

Let $\underline{a}\times\underline{b}=\underline{r}$

$\dfrac{d}{dt}(\underline{r}\times\underline{c})=\dfrac{d\underline{r}}{dt}\times\underline{c}+\underline{r}\times\dfrac{d\underline{c}}{dt}$ [Acording to hasic rule (3)]

Then, $\dfrac{d\underline{r}}{dt}=\dfrac{d(\underline{a}\times\underline{b})}{dt}=\dfrac{d\underline{a}}{dt}\times\underline{b}+\underline{a}\times\dfrac{d\underline{b}}{dt}$ [acording to basic rule (3)]

Substitute, $\dfrac{d\underline{r}}{dt}$

$$\frac{d}{dt}(\underline{a}\times\underline{b}\times\underline{c})=\frac{d}{dt}(\underline{a}\times\underline{b})\times\underline{c}+\underline{a}\times\underline{b}\times\frac{d\underline{c}}{dt}$$

$$=\left[\frac{d\underline{a}}{dt}\times\underline{b}+\underline{a}\times\frac{d\underline{b}}{dt}\right]\times\underline{c}+\underline{a}\times\underline{b}\times\frac{d\underline{c}}{dt}$$

$$=\frac{d\underline{a}}{dt}\times\underline{b}\times\underline{c}+\underline{a}\times\frac{d\underline{b}}{dt}\times\underline{c}+\underline{a}\times\underline{b}\times\frac{d\underline{c}}{dt}$$

$$\therefore\frac{d}{dt}(\underline{a}\times\underline{b}\times\underline{c})=\frac{d\underline{a}}{dt}\times\underline{b}\times\underline{c}+\underline{a}\times\frac{d\underline{b}}{dt}\times\underline{c}+\underline{a}\times\underline{b}\times\frac{d\underline{c}}{dt}$$

(iii). $\underline{r}=e^{-t}(\underline{c}_1\ \cos 2t+\underline{c}_2\ \sin 2t)$

$$\frac{d\underline{r}}{dt}=e^{-t}(c_1(-\sin 2t)\times 2+\underline{c}_2(\cos 2t)\times 2)+(c_1\cos 2t+\underline{c}_2\ \sin 2t)e^{-t}\times(-1)$$

$$\frac{d\underline{r}}{dt}=e^{-t}(-2c_1\sin 2t+2\underline{c}_2\ \cos 2t)-e^{-t}(\underline{c}_1\cos 2t+\underline{c}_2\sin 2t)$$

$$=2e^{-t}(-\underline{c}_1\sin 2t+\underline{c}_2\cos 2t)-\underline{r}$$

$$\frac{d^2\underline{r}}{dt^2}=2e^{-t}(-c_1\cos 2t\times 2+c_2(-\sin 2t)\times 2)+2(-c_1\sin 2t+\underline{c}_2\cos 2t)e^{-t}\times(-1)-\frac{dr}{dt}$$

$$\frac{d^2r}{dt^2}=-4e^{-t}(\underline{c}_1\cos 2t+\underline{c}_2\sin 2t)-2e^{-t}(-\underline{c}_1\sin 2t+\underline{c}_2\cos 2t)-\frac{dr}{dt}$$

$$\frac{d^2r}{dt^2}=-4r-\frac{dr}{dt}-r-\frac{dr}{dt}$$

$$\frac{d^2r}{dt^2}+2\frac{dr}{dt}+5r=0$$

(4) (i) $(xy+x)dy-(xy+y)dx=0$

$$x(y+1)dx-y(x+1)dx=0$$

$$\frac{(y+1)}{y}dy=\frac{(x+1)}{x}dx$$

$$\left(\frac{y}{y}+\frac{1}{y}\right)dy=\left(\frac{x}{x}+\frac{1}{x}\right)dx$$

$$\int 1dy+\int\frac{1}{y}dy=\int 1dx+\int\frac{1}{x}dx$$

$y+\ell n|y|=x+\ell n|x|+c$ $\qquad c-$is constant

(ii) $(x+1)\frac{dy}{dx}+1=2e^{-y}$

$$(x+1)\frac{dy}{dx}=(2e^{-y}-1)$$

$$\frac{1}{(2e^{-y}-1)}dy=\frac{1}{(x+1)}dx$$

$$(x+1)\frac{dy}{dx}=\left(\frac{2}{e^y}-1\right)$$

$$(x+1)\frac{dy}{dx}=\left(\frac{2-e^y}{e^y}\right)$$

$$\left(\frac{e^y}{2-e^y}\right)dy=\frac{1}{(x+1)}dx$$

$$-\ell n\left|(2-e^y)\right|=\ell n\left|(x+1)\right|+\ell n|c|$$

$$-\ell n\left|(2-e^y)\right|-\ell n\left|(x+1)\right|=\ell n|c|$$

$$(2-e^y)(x+1)=\frac{1}{c}$$

$c(x+1)(2-e^y)=1$-C is arbitrary constant.

(iii) $(e^y+2)\sin x\,dx-e^y\ \cos x\,dy=0$

$$\frac{dy}{dx}=\frac{(e^y+2)\sin x}{e^y\cos x}$$

$$\left(\frac{e^y}{e^y+2}\right)dy=\tan x\,dx$$

$$\ell n\left|e^y+2\right|=\ell n\left|\sec x\right|+c|$$

(iv) $x(y-x)\frac{dy}{dx}=y(y+x)$

$$\frac{dy}{dx}=\frac{y(y+x)}{x(y-x)}$$

Let,

$y=vx$

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

Substitude,

$$v+x\frac{dv}{dx}=\frac{v\not{x}(vx+x)}{\not{x}(vx-x)}$$

$$v+x\frac{dv}{dx}=\frac{v\not{x}(v+1)}{\not{x}(v-1)}$$

$$v+x\frac{dv}{dx}=\frac{v(v+1)}{v-1}$$

$$x\frac{dv}{dx}=\frac{v(v+1)}{v-1}-v$$

$$x\frac{dv}{dx}=\frac{v(v+1)-v(v-1)}{v-1}$$

$$x\frac{dv}{dx}=\frac{2v}{v-1}$$

$$\int\left(\frac{v}{2v}-\frac{1}{2v}\right)dv=\int\frac{1}{x}dx$$

$$\frac{1}{2}v-\frac{1}{2}\ell n|v|=\ell n|x|+c$$

$$\frac{1}{2}{y}/{x}-{1}/{2}\,\ell n\left|{y}/{x}\right|=\ell n|x|+c$$

$$\frac{1}{2}\left({y}/{x}-\ell n\left|{y}/{x}\right|\right)=\ell n|x|+c$$

(5) (i). $(x+y-10)dx+(x-y-z)dy=0$

$$g(x,y)=(x-y-2)\qquad f(x,y)=(x+y-10)$$

$$\frac{\partial g}{\partial x}=1\qquad \frac{\partial f}{\partial y}=1$$

$$\frac{\partial g}{\partial x}=\frac{\partial f}{\partial y}$$

This differential equation is exact.

General solution is, $u(x,y)$

$$u(x,y)=\int f(x,y)dx+F(y)$$

$$=\int(x+y-10)dx+F(y)$$

$$={x^2}/{2}+yx-10x+F(y)$$

$$u=\frac{x^2}{2}+yx-10x+F(y)$$

$$\frac{\partial u}{\partial y}=g(x,y)=x+F'(y)$$

$$g(x,y)=x-y-2$$

$$x+F'(y)=x-y-2$$

$$F'(y)=-(y+2)$$

$$F(y)=-\left({y^2}/{2}+2y\right)$$

$\therefore u(x,y) = x^2/2 + yx - 10x - y^2/2 - 2y + c$

(ii). $(e^y + 2)\sin x \, dx - e^y \cos x \, dy = 0$

$f(x,y) = (e^y + 2)\sin x \quad g(x,y) = -e^y \cos x$

$\frac{\partial f}{\partial y} = e^y \sin x \quad \frac{\partial g}{\partial x} = -e^y(-\sin x)$

$= e^y \sin x$

$\therefore \frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

$\therefore$ This differential equation is exact.

Solution is u(x,y)

$u(x,y) = \int f(x,y)dx + F(y)$

$= \int (e^y + 2)\sin x \; dx + F(y)$

$= e^y(-\cos x) + 2(-\cos x) + F(y)$

$= -(e^y + 2)\cos x + F(y)$

$\frac{\partial u}{\partial y} = -e^y \cos x + \frac{\partial F(y)}{\partial y}$

$\frac{\partial u}{\partial y} = g(x,y)$

$\therefore e^{-y} \cos x + \frac{\partial F(y)}{\partial y} = -e^y \cos x$

$\frac{\partial F}{\partial y} = 0$

$\therefore$ Solution is $u(x,y) = -(e^y + 2)\cos x + c$

(ii) $(xy^2 + kx^2y)dx + (x+y)x^2dy = 0$

If given differential equation is exact,

$f(x,y) = (xy^2 + kx^2y)$ $\quad g(x,y) = (x+y)x^2$

$\frac{\partial f}{\partial y} = 2xy + kx^2$ $\quad \frac{\partial g}{\partial x} = 3x^2 + 2xy$

$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

$2xy + kx^2 = 3x^2 + 2xy$

$kx^2 = 3x^2 + 2xy - 2xy$

$$k = 3$$

$$\therefore f(x,y) = xy^2 + 2x^2y \qquad\qquad g(x,y) = (x+y)x^2$$

$$u = \int f(x,y)dx + F(y)$$

$$= \int \left(xy^2 + 3x^2y\right)dx + F(y)$$

$$= \frac{x^2y^2}{2} + 3\frac{x^3}{3}y + F(y)$$

$$= \frac{x^2y^2}{2} + x^3y + F(y)$$

$$u = \frac{x^2y^2}{2} + x^3y + F(y)$$

$$\frac{\partial u}{\partial y} = \frac{x^2}{2}2y + x^3 + \frac{\partial F}{\partial y}$$

$$\frac{\partial u}{\partial y} = g(x,y)$$

$$(x+y)x^2 = x^2y + x^3 + \frac{\partial F}{\partial y}$$

$$\frac{\partial F}{\partial y} = 1$$

$$f(x,y) = 0$$

General solution is,

$$u(x,y) = \frac{x^2y^2}{2} + x^3y + c$$