MPZ 3230 – Engineering Mathematics I
Assignment No. 02 –Academic Year - 2008

Answer all questions

(1) The differential equation satisfied by a beam uniformly lorded W kg/m with one end fixed and the second end subject to tensile force P , is given by

$$EI\frac{d^2y}{dx^2} = Py - \frac{W}{2}x^2,$$

Where E is the modulus of elasticity, I is the moment of Inertia of the cross section; You can assume $\frac{P}{EI} = n^2$;

(i) Find the complementary function of the given differential equation.
(ii) Find the general solution of the given differential equation.
(iii) Show that the particular solution of the given differential equation (when $\frac{dy}{dx} = 0$ and $y = 0$ at $x = 0$) is

$$y = \frac{W}{Pn^2}(1 - Coshnx) + \frac{Wx^2}{2P}, \text{ where } n^2 = \frac{P}{EI}$$

(2)

(a) Find the particular solution of the differential equations.

(i) $x(1+y^2) - y(1+x^2)\frac{dy}{dx} = 0$ Given that $y = 0$ when $x = 0$

(ii) $(1+e^{2x})\frac{dy}{dx} + (1+y^2)e^x = 0$ Given that $y = 0$ when $x = 0$

(b). Obtain the general solutions of

(i). $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cos x$

(ii). $\frac{d^3y}{dx^3} + \frac{3d^2y}{dx^2} + 3\frac{dy}{dx} + y = e^{-x}$

(iii). using variation parameter method obtain the general solution of,

$$\frac{d^2y}{dx^2 - y} = e^{-x}\sin(e^{-x}) + \cos(e^{-x})$$

(3). (i). Using the definition of the Laplace transform, find the Laplace transforms,

(a). $f(x) = x^3 e^{-2t}$

(b). $f(t) = \sin 2t \cos 3t$

(c). $f(x) = \begin{cases} x^2 & 0 < x < 2 \\ x-1 & 2 < x < 3 \\ 7 & x > 3 \end{cases}$

(d). $\dfrac{2(3S^2-1)}{(S^2+1)^3}$

(ii). Find the inverse Laplace transforms of the

(a). $\dfrac{S-4}{4(S-3)^2+16}$

(b). $\dfrac{S+2}{S^2-2S-8}$

(c). $\dfrac{S+1}{(S^2+6S+13)^2}$

(iii). Solve the following initial value problem $y''+4y'+3y=t \quad t>0$; by using the Laplace transform method, given that $y(0)=0$ and $y'(0)=1$,

(4). (a). If $P+q+r=0$

Prove that,

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

(b). $A = \begin{bmatrix} 3 & -6 & -3 \\ 3 & -3 & -5 \\ 0 & 4 & -8 \end{bmatrix}$

Find A^{-1}

(c). Find x,y,z using Cramer's rule

$2x - y + 3z = 9$

$x + y + z = 6$

$x - y + z = 2$

(5) (a). Define the determinant of a 3x3 matrix A and B are two square matrices and I is the identity matrix. Assuming the results.

$$AA^{-1} = I \quad and \quad |AB| = |A||B|,$$

Show that $|A^{-1}| = \dfrac{1}{|A|}$

Show that $\begin{vmatrix} a-x & a-y & a-z \\ b-x & b-y & b-z \\ c-x & c-y & c-z \end{vmatrix} = 0$

(b). The 3 currents in a electrical circuit are given by following equations.

$$i_1 + 2i_2 + 3i_3 = 1$$

$$i_1 + 3i_2 + 5i_3 = 3$$

$$i_1 + 5i_2 + 12i_3 = -4$$

(i). write these equations in matrix form.

(ii). Using matrix inversion method, calculate the three currents of the electrical circuits.

(6). (i). If $Z = x^4 + 2x^2y + y^3$ and $x = t\cos\theta$ and $x = t\cos\theta$ and $y = t\sin\theta$

Find $\dfrac{\partial z}{\partial t}$, and $\dfrac{\partial z}{\partial \theta}$ in their simplest forms.

(ii). If $Z = x\ell n(x^2 + y^2) - 2y\tan^{-1}\left(\dfrac{y}{x}\right)$ verify that,

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + 2x$$

(iii). (a).show that $P \cap q \equiv q \cap p, \quad p \cup q \equiv q \cup p$

(b).show that $(p \cup q) \cup r \equiv p \cup (q \cup r)$, $(p \cap q) \cap r \equiv p \cap (q \cap r)$

(c).show that

$$(p \cap q) \cup r \equiv (p \cup r) \cap (q \cup r), and (p \cup q) \cap r \equiv (p \cap r) \cap (q \cup r)$$

Please send your assignment **on or before the due date** to the following address. **Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.**

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(1).

(i) $EI\dfrac{d^2y}{dx^2} = py - \dfrac{\omega x^2}{2}$

$$\left\{\frac{d^2y}{dx^2} - \frac{py}{EI}\right\} = -\frac{\omega x^2}{2EI}$$

$$\left[D^2 - n^2\right]y = \frac{\omega x^2}{2EI} \quad \text{, Where } P/_{EI} = n^2$$

$\therefore$ Auxilary equation $D^2 - n^2 = 0 \therefore D = \pm n$

$\therefore$ Compementary function $= C_1e^{nx} + C_2e^{-nx}$

(ii)Particular Interregnal PI,

$$y = \frac{1}{(D^2 - n^2)}\left\{\frac{-\omega x^2}{2EI}\right\} = \frac{-\omega}{2EI}\left\{\frac{1}{D^2 - n^2}x^2\right\}$$

$$\frac{1}{D^2 - n^2} = \frac{-1}{n^2 - D^2} = \frac{1/_{n^2}}{\left[1 - (D/_n)^2\right]}$$

$$= 1/_{n^2}\left[1 - (D/_n)^2\right]^{-1}$$

$$= 1/_{n^2}\left[1 + \frac{D^2}{n^2} + \frac{D^4}{n^4} + \frac{D^6}{n^6} + ..\right]$$

$$\therefore \text{P.I.} = \text{y} = \frac{\omega}{2\text{n}^2EI}\left[1 + \frac{D^2}{n^2} + \frac{D^4}{n^4} + ..\right]\{x^2\}$$

$$= \frac{\omega}{2n^2EI}\left(x^2 + 2/_{n^2}\right)$$

The general solution is $y = CF + PI$

$$= C_1e^{nx} + C_2e^{-nx} + \frac{\omega}{2n^2EI}\left(x^2 + 2/_{n^2}\right)$$

(ii)Given that, $\dfrac{dy}{dx} = 0, y = 0$ when $x = 0$,

$$\frac{dy}{dx} = C_1ne^{nx} + -nc_2e^{-nx} + \frac{\omega x}{n^2EI}$$

$$0 = (C_1 - C_2)n$$

$$\therefore C_1 = C_2$$

When y=0;

$$0 = C_1 + C_2 + \frac{\omega}{EIn^4}$$

$$\therefore C_1 = C_2 = -\frac{\omega}{2n^4EI}$$

Substitute the values, of C_1 and C_2 to the general solution;

$$y = \frac{-\omega}{2n^4EI}\left(e^{nx} + e^{-nx}\right) + \frac{\omega}{2n^2EI}\left(x^2 + \frac{2}{n^2}\right)$$

Given $P/EI = n^2$;

$$y = \frac{-\omega}{n^2p}\left(\frac{e^{nx} + e^{-nx}}{2}\right) + \frac{\omega}{2p}\left(x^2 + \frac{2}{n^2}\right)$$

$$= \frac{-\omega}{n^2p}\cos h\,(nx) + \frac{\omega x^2}{2p} + \frac{\omega}{n^2p}$$

$$y = \frac{\omega}{n^2p}\left(1 - \cosh\,(nx)\right) + \frac{\omega x^2}{2p}$$

(2). (a) (ii) $x\left(1+y^2\right) - y\left(1+x^2\right)\frac{dy}{dx} = 0$ Given y=0, when x=0

$$\frac{dy}{dx} = \frac{x\left(1+y^2\right)}{y\left(1+x^2\right)}$$

$$\frac{y}{\left(1+y^2\right)}dy = \frac{x}{\left(1+x^2\right)}dx$$

$\frac{1}{2}\int\frac{2y}{1+y^2}dy = \frac{1}{2}\int\frac{2x}{\left(1+x^2\right)}dx + C.$ (C- is any arbitrary constant)

$$\frac{1}{2}\ell n\left|1+y^2\right| = \frac{1}{2}\ell n\left|1+x^2\right| + C$$

$y = 0$ When x=0, then $\frac{1}{2}\ell n1 - \frac{1}{2}\ell n1 = c; c = 0$

$\therefore$ The particular Solution; $\frac{1}{2}\ell n\frac{1+y^2}{1+x^2} = 0$

$$x^2 - y^2 = 0$$

(ii) $\left(1+e^{2x}\right)\frac{dy}{dx} + \left(1+y^2\right)e^x = 0$ Given, y=0, when x=0.

$$\frac{dy}{dx} = -\frac{(1+y^2)e^x}{(1+e^{2x})}$$

$$\int\frac{1}{(1+y^2)}dy = \int\frac{e^x}{(1+e^{2x})}dx$$

$$\tan^{-1}(y) = \int\frac{e^x}{1+e^{2x}}dx + C ----(1)$$

$$\int\frac{e^x}{1+e^{2x}}dx \quad \text{Let} \quad t = e^x; \text{then}. \frac{dt}{dx} = e^x$$

$$\therefore \int\frac{e^x}{1+e^{2x}}dx = \int\frac{t}{1+t^2}\times\frac{1}{t}dt = \int\frac{1}{1+t^2}dt = \tan^{-1}(t)$$

Substitute, $t = e^x$; $\int\frac{e^x}{1+e^{2x}}dx = \tan^{-1}e^x$

Now, Substitute the values of equation,

$$\tan^{-1}(y) = -\tan^{-1}(e^x) + C$$

Given y=0, when x=0

$$\tan^{-1}(0) = -\tan^{-1}(e^0) + C$$

$$C = 0 + \tan^{-1}1 = \pi/4$$

$\therefore$ Particular solution is; $\tan^{-1}(y) = -\tan^{-1}(e^x) + \pi/4$

$$\tan^{-1}(y) + \tan^{-1}(e^e) = \pi/4$$

(b) (i) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cos x$

The characteristic equation is,

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$$

$$m^2 + 4m + 5 = 0$$

$$m = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-4\pm\sqrt{4^2-4.1.5}}{2} = \frac{-4\pm(-4)^{\frac{1}{2}}}{2} = -2\pm \underline{i}$$

$$m_1 = 2+\underline{i} \qquad m_2 = -2-i$$

There for complementary function $y_c = e^{-2x}(A\cos 2x + B\sin 2x)$ A and B are constants.
In order to find particular Integral, use trial function method,

Let, $y_p = \alpha\cos x + \beta\sin x$ (α and β are unknown constants)

$$\therefore \frac{dy}{dx} = -\alpha \sin x + \beta \cos x$$

$$\frac{d^2 y_p}{dx^2} = -\alpha \cos x - \beta \sin x$$

Substitute;

$$\frac{d^2 y_p}{dx^2} + 4\frac{dy_p}{dx} + 5y_p = -2\cos x$$

$$-\alpha \cos x - \beta \sin x + 4(-\alpha \sin x + \beta \cos x) + 5(\alpha \cos x + \beta \sin x) = -2\cos x$$

$$(4\alpha + 4\beta)\cos x + (4\beta - 4\alpha)\sin x = -2\cos x$$

Equating terms; $(4\alpha + 4\beta) = -2; 8\alpha = -2; \alpha = -\frac{1}{4}$

$$4\beta - 4\alpha = 0; \alpha = \beta; \beta = -\frac{1}{4}$$

$$\alpha = -\frac{1}{4} \qquad \beta = -\frac{1}{4}$$

$\therefore$ Particular integral is $-\frac{1}{4}\cos x - \frac{1}{4}\sin x$

$\therefore$ The general solution is $y = e^{-2x}(A\cos x + B\sin x) - \frac{1}{4}(\cos x + \sin x)$

(ii). $\frac{d^3 y}{dx^3} + 3\frac{d^2 y}{dx^2} + 3\frac{dy}{dx} + y = -e^x$

The characteristic equation is,

$$m^3 + 3m^2 + 3m + 1 = 0$$

$$(m+1)^3 = 0$$

$$m_1 = -1, \quad m_2 = -1, \quad m_3 = -1$$

Therefore the complementary function is,

$y_c = Ae^{-x} + Bxe^{-x} + Cx^2 e^{-x}$ A, B, C are arbitrary constants.

In order to find the particular integral, Using D operator method,

$$y_p = \frac{1}{(D+1)^3} e^{-x} dx$$

$$= \frac{x^3!}{3!} e^{-x} = \frac{x^3}{3 \times 2} e^{-x}$$

$$= \frac{x^3 e^{-x}}{6}$$

Since particular Integral is $=\frac{x^3}{6}e^{-x}$

General Solution is

$$y = Ae^{-x} + Bxe^{-x} + Cx^2e^{-x} + \frac{x}{6}e^{-x}$$

$$y = \left(A + Bx + Cx^2 + \frac{x^3}{6}\right)e^{-x} \qquad A, B, C \text{ are arbitary constant}$$

(ii). $\frac{d^2y}{dx^2} - y = e^{-x}\sin(e^{-x}) + \cos(e^{-x})$

Complementary function; $\frac{d^2y}{dx^2} - y = 0$

Characteristic equation; $m^2 - 1 = 0$

Roots of the characteristic equation $m_1 = 1, m_2 = -1$

Complementary function is $y_c = Ae^{x} + Be^{-x}$ Where A and B are arbitrary constant.

$$\therefore y_1 = e^{x} \text{ and } y_2 = e^{-x}$$

Particular Integral, using variation parameter method,

The wranstian $\omega(x) = y_1y_2^1 - y_2y_1^1$

$$= e^{x}(-e^{-x}) - e^{-x}e^{-x} = -1 - 1 = -2$$

$$\int\frac{y_1 f(x)}{\omega(x)}dx = -\frac{1}{2}\int e^{x}\{e^{-x}\sin(e^{-x}) + \cos(e^{-x})\}dx$$

$$= -\frac{1}{2}(\int\sin e^{-x}dx + \int e^{x}\cos e^{-x}dx)$$

$$= -\frac{1}{2}(\int\sin e^{-x}dx + \int\cos e^{-x} \times \frac{d}{dx}(e^{-x})dx)$$

$$= -\frac{1}{2}(\int\sin e^{-x}dx + e^{x}\cos e^{-x} - \int e^{x}(-\sin e^{-x})(-e^{-x})dx)$$

$$= -\frac{1}{2}(\int\sin e^{-x}dx + e^{x}\cos e^{-x} - \int\sin e^{-x}dx)$$

$$= -\frac{1}{2}e^{x}\cos e^{-x}$$

$$\int\frac{y_2 f(x)}{\omega(x)} = -\frac{1}{2}(\int e^{-x}\{e^{-x}\sin(e^{-x}) + \cos(e^{-x})\}dx)$$

$$= -\frac{1}{2}(\int(e^{-2x}\sin(e^{-x}) + e^{-x}\cos e^{-x})dx)$$

$$= -\frac{1}{2}\int e^{-2x}\sin e^{-x}dx - \frac{1}{2}\int \cos e^{-x} \times \frac{de^{-x}}{dx} \times dx$$

$$= -\frac{1}{2}\int e^{-2x}\sin e^{-x}dx - \frac{1}{2}\cos e^{-x}\left(-e^{-x}\right) - \frac{1}{2}\int -e^{-x}\left(-\sin e^{-x}\right)\left(-e^{-x}\right)dx$$

$$= -\frac{1}{2}\int e^{-2x} \times \sin e^{-x}dx - \frac{e^{-x}}{2}\cos e^{-x} \times -\frac{1}{2}\int e^{-2x}\sin e^{-x}dx$$

$$= -\int e^{-2x}\sin e^{-x}dx - \frac{e^{-x}}{2}\cos e^{-x}$$

Take $\int e^{-2x}\sin e^{-x}dx$, and Let $e^{-x} = t$

$$e^{-x}(-1) = \frac{dt}{dx}$$

Substitute, $\int \alpha \times \alpha \times \sin\alpha \times \frac{d\alpha}{(-\alpha)} \quad = \int -\alpha\sin\alpha \, d\alpha$

$$= -\int \alpha \frac{d}{d}(-\cos\alpha)dx$$

$$= -\left\{\alpha(-\cos\alpha) - \int -\cos\alpha \times 1 \times d\alpha\right\}$$

$$= \alpha\cos\alpha + \int \cos\alpha \, d\alpha$$

$$= \alpha\cos\alpha - \sin\alpha$$

Substitute, $\alpha = e^{-x}; = e^{-x}\cos e^{-x} - \sin e^{-x}$

$$\int \frac{y_2 f(x)}{\omega(x)} = -(e^{-x}\cos e^{-x} - \sin e^{-x}) + \frac{e^{-x}}{2}\cos e^{-x}$$

$$\therefore y_p = \frac{y_1(x)}{a_0}\int \frac{y_2(x)f(x)}{\omega(x)}dx - \frac{y_2(x)}{a_0}\int \frac{y_1(x)f(x)}{\omega(x)}dx$$

$$= e^{x}\left\{-\frac{1}{2}e^{-x}\cos e^{-x} + \sin e^{-x}\right\} - e^{-x}\left\{-\frac{1}{2}e^{x}\cos e^{-x}\right\}$$

$$= -\frac{1}{2}\cos e^{-x} + e^{x}\sin e^{-x} + \frac{1}{2}\cos e^{-x}$$

$$y_p = e^{x}\sin e^{-x}$$

General Solution; $= y_c + y_p$

$$= \underline{\underline{Ae^{x} + Be^{-x} + e^{x}\sin e^{x}}}$$

(3). (i).(a) $f(x) = x^3 e^{-2x}$

$$L[f(x)] = \int_0^\infty f(x) \times e^{-sx}dx = \int_0^\infty x^3 e^{-2x} \times e^{-sx}dx$$

$$= \int_0^\infty x^3 e^{-(s+2)x} dx$$

$$= [x^3(-1)\frac{1}{s+2}.e^{-x(s+2)}]_0^\infty + \int_0^\infty \frac{e^{-x(s+2)}}{(s+2)^2} \times 6x dx$$

$$= \left[(-1).\frac{e^{-x(s+2)}}{(s+2)^3}.6x\right]_0^\infty + \int_0^\infty \frac{e^{-x(s+2)}}{(s+2)} \times 6 dx$$

$$= \left[\frac{-e^{-x(s+2)}}{(s+2)^4}.6\right]_0^\infty = \frac{6}{(s+2)^4}$$

$\therefore$ The Laplace transform of $x^3 e^{-2x}$ is $= \frac{6}{(s+2)^4}$

(b) $f(t) = \sin 2t \cos 3t$

$$L[f(t)] = \int_0^\infty (\sin 2t \cos 3t) e^{-st} dt$$

$$= \int_0^\infty \frac{1}{2}\{\sin 5t + \sin(-t)\} e^{-st} \times dt$$

$$= \frac{1}{2}\int_0^\infty e^{-st}.\sin 5t - \frac{1}{2}\int_0^\infty e^{-st}.\sin t dt$$

$$= \frac{5}{(s^2+25)} - \frac{1}{2}\left[\frac{1}{s^2+1}\right]$$

$$\therefore L(f(t)) = \frac{1}{2}\left\{\frac{5s^2+5-s^2-5^2}{(s^2+5^2)(s^2+1)}\right\} = \frac{1}{2}\left[\frac{4s^2-20}{(s^2+5^2)(s^2+1)}\right]$$

$$= \frac{2}{}\left(\frac{(s^2-5)}{(s^2+25)(s^2+1)}\right)$$

(e) $f(x) = \begin{Bmatrix} x^2 & 0 < x < 2 \\ x-1 & 2 < x < 3 \\ 7 & x > 3 \end{Bmatrix}$

$$L(f(x)) = \int_0^\infty f(x) \times e^{-sx} dx$$

$$L(f(x)) = \int_0^2 x^2 e^{-sx} dx + \int_2^3 (x-1) e^{-sx} dx + \int_3^\infty 7 e^{-sx} dx$$

$$= \left[\frac{x^2 e^{-sx}}{-s}\right]_0^2 + \frac{2}{s}\int_0^2 x.e^{-sx} dx - \left[\frac{xe^{-sx}}{s}\right]_2^3 - \left[\frac{e^{-sx}}{s^2}\right]_2^3 + \left[\frac{e^{-sx}}{s}\right]_2^3 - 7\left[\frac{e^{-sx}}{s}\right]_3^\infty$$

$$= \frac{-4e^{-2s}}{s} + 0 + \frac{2}{s}\left(\left[\frac{xe^{-sx}}{s}\right]_0^2 - \left[\frac{e^{-sx}}{s^2}\right]_0^2\right) - \frac{3e^{3s}}{s} + \frac{2e^{-2s}}{s} - \frac{e^{-3s}}{s^2} + \frac{e^{-2s}}{s^2} + \frac{e^{-3s}}{s} - \frac{e^{-2s}}{s} + 7\frac{e^{-3s}}{s}$$

$$= \frac{1}{s}\left[-3e^{-2s} + 5e^{-3s}\right] - \frac{3e^{-2s}}{s^2} - \frac{2}{s^3}(e^{-2s} - 1) + \frac{7e^{-3s}}{s}$$

$$= -\frac{e^{-2s}}{s^3}(2 + 3s + 3s^2) + \frac{e^{-3s}}{s^2}(5s - 1) + \frac{2}{s^3}$$

(ii). (a). $\dfrac{5-4}{4(5-3)^2+16} = \dfrac{1}{4}\left[\dfrac{s-4}{(s-3)^2+4}\right]$

$$= \frac{1}{4}\left\{\frac{s-3-1}{(s-3)^2+2^2}\right\} = \frac{1}{4}\left\{\frac{s-3}{(s-3)^2+2^2} - \frac{1}{(s-3)^2+2^2}\right\}$$

$$= \frac{1}{4}\left\{\frac{(s-3)}{(s-3)^2+2^2}\right\} - \frac{1}{4}\left\{\frac{1}{(s-3)^2+2^2}\right\}$$

$$\therefore L^{-1}\left\{\frac{(s-4)}{4(s-3)^2+16}\right\} = \frac{1}{4}L^{-1}\left\{\frac{(s-3)}{(s-3)^2+2^2}\right\} - \frac{1}{4}L^{-1}\left\{\frac{1}{(s-3)^2+2^2}\right\}$$

$$= \frac{1}{4}e^{3t}\cos 2t - \frac{1}{4}e^{3t}\sin 2t$$

$$= \frac{1}{4}e^{3t}\{\cos 2t - \sin 2t\}$$

(b). $\dfrac{s+2}{s^2-2s-8} = \dfrac{(s+2)}{(s-4)(s+2)} = \dfrac{1}{(s-4)}$

$$L^{-1}\left\{\frac{s+2}{s^2-2s-8}\right\} = L^{-1}\left\{\frac{1}{(s-4)}\right\} = \underline{\underline{e^{4t}}}$$

(c) $\dfrac{s+1}{(s^2+6s+13)^2} = \dfrac{s+3-2}{\{(s+3)^2+(2)^2\}^2} = \dfrac{s+3}{\{(s+3)^2+(2)^2\}^2} - \dfrac{2}{\{(s+3)^2+(2)^2\}^2}$

$$= \frac{(s+3)}{[(s+3)^2+2^2][(s+3)^2+2^2]} - \frac{2}{[(s+3)^2+2^2][(s+3)^2+2^2]}$$

$$L^{-1}\left\{\frac{s+1}{(s^2+6s+13)^2}\right\} = \frac{(s+3)}{[(s+3)^2+2^2]} \times \frac{1}{[(s+3)^2+2^2]} - \frac{2}{[(s+3)^2+2^2]} \times \frac{1}{[(s+3)^2+2^2]}$$

$$= e^{-3t} \times \cos 2t \times e^{-3t}\sin 2t - 2\{e^{-3t} \times \sin 2t \times e^{-3t}\sin 2t\}$$

$$= e^{-6t}\sin 2t\,\cos 2t - 2e^{-6t}\sin^2 2t$$

$$= \underline{\underline{e^{-6t}\sin 2t\,\{\cos 2t - \sin 2t\}}}$$

(iii). $y''+4y'+3y=t \quad y(0)=0; y'(0)=1$

Using Laplace Transformation method,

$$\{s^2y(s)-sy(0)-y'(0)\}+4\{sy(s)-y(0)\}+3y(s)=\frac{1}{s^2}------(1)$$

But given; $y(0)=0, y'(0)=1,$

Substitute given values for equation (1),

$$\{S^2y(s)-0-1\}+4\{sy(s)-0\}-3y(s)=\frac{1}{s^2}$$

$$(s^2+4s-3)y(s)-1=\frac{1}{s^2}$$

$$y(s)=\frac{1+s^2}{(s^2+4s+3)}=\frac{AS+B}{s^2}+\frac{C}{(s+3)}+\frac{D}{(s+1)}$$

$$1+s^2=(As+B)(s^2+4s+1)+cs^2(s+1)+Ds^2(s+3)$$

$$B=\frac{1}{3}; A=\frac{-4}{9}; D=1; C=\frac{-5}{9}$$

$$y(s)=\frac{\frac{4}{9}s+\frac{1}{3}}{s^2}+\frac{\frac{5}{9}}{s-3}-\frac{1}{s-1}$$

$$=\frac{4}{9s}+\frac{1}{3s^2}+\frac{5}{9}\frac{1}{s-3}-\frac{1}{(s-1)}$$

$$L^{-1}[y(s)]=L^{-1}\left\{\frac{4}{9s}+\frac{1}{3s^2}+\frac{5}{9}\frac{1}{s-3}-\frac{1}{(s-1)}\right\}$$

$$y(t)=\frac{4}{9}+\frac{1}{3}t+\frac{5}{9}e^{3t}-e^{t}$$

(4)(a) $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$

L: H: S: $\begin{vmatrix} pa & ab & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pa(rqa^2-p^2cb)-qb(q^2ac-prb^2)+rc(qpc^2-r^2ab)$

$$=pqra^3-p^3abc-q^3abc+pqrb^3+pqrc^3-r^3abc$$

$$=pqr(a^2+b^3+c^3)-abc(p^3+q^3+r^3)$$

Given, $p+q+r=0 \therefore r=-(p+q)$

Substitute, $r = -(p+q)$

$$= pqr\left(a^3+b^3+c^3\right) - abc\left(p^3+q^3+(-p-q)^3\right)$$
$$= pqr\left(a^3+b^3+c^3\right) - abc\left(p^3+q^3-(p^3+q^3+3p^2q+3pq^2)\right)$$
$$= pqr\left(a^3+b^3+c^3\right) - abc(-3pq(p+q))$$
$$= pqr\left(a^3+b^3+c^3\right) - abc(-3pq(-r))[\because p+q=-r]$$
$$= pqr\left(a^3+b^3+c^3-3abc\right)$$

$$R:H:S \Rightarrow = pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

$$= pqr\left[a\left(a^2-bc\right) - b\left(ac-b^2\right) + c\left[c^2-ab\right]\right]$$
$$= pqr\left[a^3+b^3+c^3-3abc\right]$$

$\therefore R:H:S := L:H:S$

Therefore,

$$\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

(b) $A = \begin{bmatrix} 3 & -6 & -3 \\ 3 & -3 & -5 \\ 0 & 4 & -8 \end{bmatrix}$

Row Transformation method,

$$A = IA$$

$$\begin{bmatrix} 3 & -6 & -3 \\ 3 & -3 & -5 \\ 0 & 4 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\downarrow R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 3 & -6 & -3 \\ 0 & 3 & -2 \\ 0 & 4 & -8 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\downarrow R_1 \to R_1/3$$

$$\begin{bmatrix} 1 & -2 & -1 \\ 0 & 3 & -2 \\ 0 & 4 & -8 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\downarrow R_1 \to R_1 + R_3/2$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 3 & -2 \\ 0 & 4 & -8 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 1/2 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\downarrow R_2 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 4 & -8 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 1/2 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\downarrow R_3 \to R_3 - 4R_2$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 0 & 16 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 1/2 \\ 1 & -1 & 1 \\ 4 & 4 & -3 \end{bmatrix} A$$

$$\downarrow R_3/16$$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/3 & 0 & 1/2 \\ 1 & -1 & 1 \\ -1/4 & 1/4 & -3/16 \end{bmatrix} A$$

$$\downarrow R_1 \to R_1 + 5R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11/12 & 5/4 & -7/16 \\ 1 & -1 & -1 \\ -1/4 & 1/4 & -3/16 \end{bmatrix} A$$

$$\downarrow R_2 \to R_2 + 6R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11/12 & 5/4 & -7/16 \\ -1/2 & 1/2 & -1/8 \\ 1/4 & 1/4 & -3/16 \end{bmatrix} A$$

$$I = A^{-1} A$$

$$\therefore \text{Inverse metrix is} = \begin{bmatrix} 11/12 & 5/4 & -7/16 \\ -1/2 & 1/2 & -1/8 \\ 1/4 & 1/4 & -3/16 \end{bmatrix}$$

(c)
$$\begin{aligned} 2x - y + 3x &= 9 \\ x + y + z &= 6 \\ x - y + z &= 2 \end{aligned}$$

In Matrix form;

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 6 \\ 2 \end{bmatrix}$$

$$Ax = B$$

det A= $2(1-(-1)) - (-1)(1-1) + 3(-1-1) = -2$

According to Cramer's rule,

$$x = \frac{\begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{9(1-(-1)) + 1(6-2) + 3(-6-2)}{-2}$$

$$= \frac{-2}{-2} = 1 \qquad \underline{\underline{x = 1}}$$

$$y = \frac{\begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}} = \frac{2(6-2) - 9(1-1) + 3(2-6)}{-2}$$

$$= \frac{-4}{-2} = 2$$

$\underline{\underline{y = 2}}$

$Z = 3$;

$\underline{\underline{x = 1; y = 2; z = 3}}$

(5)Given; $AA^{-1} = I and |AB| = |A||B|$

$|AA^{-1}| = |I|$; $\qquad |AA^{-1} = 1|$

If $|AB| = |A||B|$; $\quad |AA^{-1}| = |A||A^{-1}|$

$1 = |A||A^{-1}|$

$$|A^{-1}| = \frac{1}{|A|}$$

(b)

$$\begin{vmatrix} a-x & a-y & a-z & a-p \\ b-x & b-y & b-z & b-p \\ c-x & c-y & c-z & c-p \\ d-x & d-y & d-z & d-p \end{vmatrix}$$

$$\downarrow R_1 \to R_1 - R_2$$
$$R_2 \to R_2 - R_3$$

$$\begin{vmatrix} a-b & a-b & a-b & a-b \\ b-c & b-c & b-c & b-c \\ c-x & c-y & c-z & c-p \\ d-x & d-y & d-z & d-p \end{vmatrix}$$

$$\downarrow$$

$$(a-b)(b-c)\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ c-x & c-y & c-z & c-p \\ d-x & d-y & d-z & c-p \end{vmatrix}$$

$$\downarrow R_1 \to R_1 - R_2$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ c-x & c-y & c-z & c-p \\ d-x & d-y & d-z & c-p \end{vmatrix}$$

If all elements in any row (column) of a determinant are zero, then the value of the determinant is zero.

$$\therefore \begin{vmatrix} a-x & a-y & a-z & a-p \\ b-x & b-y & b-z & b-p \\ c-x & c-y & c-z & c-p \\ d-x & d-y & d-z & d-p \end{vmatrix} = 0$$

(b).
$$i_1 + 2i_2 + 3i_3 = 1$$
$$i_1 + 2i_2 + 5i_3 = 3$$
$$i_1 + 5i_2 + 12i_3 = -4$$

In Matrix form;

$$\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 5\\ 1 & 5 & 12\end{bmatrix}\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix}=\begin{bmatrix}1\\ 3\\ -4\end{bmatrix}$$

$$Ax = B,$$

$$Ax = B$$

$$A^{-1}Ax = A^{-1}B$$

$$Ix = A^{-1}B$$

$$x = A^{-1}B \text{------} (1)$$

$$A^{-1} = \frac{Adj\ A}{|A|}$$

$$|A| = \begin{vmatrix}1 & 2 & 3\\ 1 & 3 & 5\\ 1 & 5 & 12\end{vmatrix} = 1(3\times 12-25)-2(12-5)+3(5-3) = 3$$

Coefficient of (A);

$$cof_{11} = (-1)^{1+1}\begin{vmatrix}3 & 5\\ 5 & 12\end{vmatrix} = (36-25) = 11$$

$$cof_{12} = (-1)^{1+2}\begin{vmatrix}1 & 5\\ 1 & 12\end{vmatrix} = (-1)(12-5) = -7$$

$$cof_{13} = 2\ ;\ cof_{21} = -9; cof_{22} = 9; cof_{23} = -3; cof_{31} = 1; cof_{32} = -2; cof_{33} = 1$$

$$\therefore \text{Coefficent of A} = \begin{vmatrix}11 & -7 & 2\\ -9 & 9 & -3\\ 1 & -2 & 1\end{vmatrix}$$

$$\therefore \text{Adj(A)} = [cof(A)]^T = \begin{vmatrix}11 & -9 & 1\\ -7 & 9 & -2\\ 2 & -3 & 1\end{vmatrix}$$

$$A^{-1} = \begin{bmatrix}11/3 & -9/3 & 1/3\\ -7/3 & 9/3 & -2/3\\ 2/3 & 3/3 & 1/3\end{bmatrix}$$

By (1); $x=A^{-1}B$

$$\begin{bmatrix}i_1\\ i_2\\ i_3\end{bmatrix} = \begin{bmatrix}11/3 & -9/3 & 1/3\\ -7/3 & 9/3 & -2/3\\ 2/3 & 3/3 & 1/3\end{bmatrix}\begin{bmatrix}1\\ 3\\ -4\end{bmatrix}$$

$$i_1 = 11/3 - 9 - 4/3 = -20/3 = \underline{\underline{-6.66A}}$$

$$i_2 = 7^{1}1/3 + 9 - 8/3 = -26/3 = \underline{\underline{-9.33A}}$$

$$i_3 = 2/3 - 3 - 4/3 = -11/3 = \underline{\underline{-3.66A}}$$

(6) (a) $p \cap q \equiv q \cap p$ $\qquad$ $p \cup q \equiv q \cup p$

P	Q	$p \cap q$	$q \cap p$	$p \cup q$	$q \cup p$
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	0	1	1
1	1	1	1	1	1

$\therefore \underline{\underline{p \cap q \equiv q \cap p}}$ $\qquad$ $\underline{\underline{p \cup q \equiv q \cup p}}$

(b). $\underline{\underline{(p \cup q)}} \cup r \equiv p \cup (q \cup r)$

P	Q	R	$\underline{\underline{(p \cup q)}}$	$\underline{\underline{(p \cup q)}} \cup r$	$(q \cup r)$	$p \cup (q \cup r)$
0	0	0	0	0	0	0
0	0	1	1	1	1	1
0	1	0	1	1	1	1
1	0	0	1	1	0	1
1	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	1	1	1	1	1
1	1	1	1	1	1	1

$\underline{\underline{(p \cup q) \cup r \equiv p \cup (q \cup r)}}$

$(p \cap q) \cap r \equiv p \cap (q \cap r)$

P	Q	R	$(p \cap q)$	$(q \cap r)$	$(p \cap q) \cap r$	$p \cap (q \cap r)$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
1	0	0	0	0	0	0
1	1	0	1	0	0	0
0	1	1	0	1	0	0
1	0	1	0	0	0	0
1	1	1	1	1	1	1

$\underline{\underline{(p \cap q) \cap r \equiv p \cap (q \cap r)}}$

(c) $(p \cap q) \cup r \equiv (p \cup r) \cap (q \cup r)$

P	Q	R	$(p \cap q)$	$(p \cup r)$	$(q \cup r)$	$(p \cap q) \cup r$	$(p \cup r) \cap (q \cup r)$
0	0	0	0	0	0	0	0
0	0	1	0	1	1	1	1
0	1	0	0	0	1	0	0
1	0	0	0	1	0	0	0
1	1	0	1	1	1	1	1
0	1	1	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1

$\underline{\underline{(p \cap q) \cup r \equiv (p \cup r) \cap (q \cup r)}}$

$(p \cup q) \cap r \equiv (p \cap r) \cup (q \cap r)$

P	Q	R	$(p \cup q)$	$(p \cap r)$	$(q \cap r)$	$(p \cup q) \cap r$	$(p \cap r) \cup (q \cap r)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0
1	0	0	1	0	0	0	0
1	1	0	1	0	0	0	0
0	1	1	1	0	1	1	1
1	0	1	1	1	0	1	1
1	1	1	1	1	1	1	1

$\underline{\underline{(p \cup q) \cap r \equiv (p \cap r) \cup (q \cap r)}}$