MPZ 3230 – Engineering Mathematics I

Assignment No. 03 –Academic Year - 2008

Answer all questions

(1). Newton's in an atomic pile increase at the rate proportional to the number of neutrons present at any instant. If no Neutrons are initially present and $N_1 and N_2$ Neutrons are present at time $T_1 and T_2$ respectively

Show that,

$$\left(\frac{N_2}{N_0}\right)^{T_1} = \left(\frac{N_1}{N_0}\right)^{T_2}$$

N (t) is number of neutrons at any time t.

(2) OPQR is a tetrahedron such that OP = OQ = OR = a and $P\hat{Q}R = \pi/2$ .N is the mid point of PR.

- (i) Prove that ON is perpendicular to the plane PQR.
- (ii) Find the angle between the planes PQR and OPQ, when the triangle PQR is isosceles and PQ = a.
- (iii) Find also the angle between the planes OPQ an OQR

(3) (a). Find the root of the following equations, using the Bisection method correct to four decimal places.

(i). $f(x) = x^3 - 5x + 1$ which lies between 2 & 3.

(b). Find real root of the following equations correct four decimal places by the method of false position,

(i). $x^3 - 3x + 4 = 0$

(ii). $x^3 + x - 1 = 0, near\ x = 1$

(c). this quadratic has nearly equal roots:

$(x-1.99)(x-2.01) = x^2 - 4x + 3.9999$

- (a) Use Newton's method with a starting value of 2.1. Is there quadratic convergence?
- (b) Use Newton's with a starting value of 1.9. Is there quadratic convergence?
- (c) Use Newton's with a starting value of 2.0. Is there quadratic convergence?
- (d) Repeat part (c), but use a quadratic that has roots at x = 2.01 and 2.03, starting with x = 2.02. If the results are different, explain.

(4) (a). (i). Write the Gause seidel and Jacobi algorithms.

(ii).

$$5x + 2y + z = 12$$
$$x + 4y + 2z = 15$$
$$x + 2y + 5z = 20$$

- (a) Solve by Jacobi method.
- (b) Solve by Gause – seidel method,

Compare two method and comment on which method is most suitable?

(b) Using Gauss-seidal iteration method, solve the system of equations,

$$10x-2y-z-w=3$$
$$-2x+10y-z-w=15$$
$$-x-y+10z-2w=27$$
$$-x-y-2z+10w=-9$$

(5)(a). solve the following equation using Gauge elimination method,

$$10x-7y-3z+5u=6$$
$$-6x+8y-z-4u=5$$
$$5x-2y+9z+4u=7$$
$$3x+y+4z+11u=2$$

(c) Find the inverse of the matrix

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$$ Using Gauss elimination method (Hint: use matrix from in this method)

(c). Apply LU Factorisation method, solve,

$$2x-y+z=5$$
$$2y-z+u=1$$
$$x+2z-u=8$$
$$x+y+2u=2$$

Please send your assignment **on or before 12/11/2008** to the following address. **Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.**

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MPZ3230 – Engineering Mathematics I
Assignment No. 03 – 2008
Model Answer

(01) Let N(t) – Number of neutrons present at any time t.

$$\frac{dN(t)}{dt} \propto N(t)$$

$$\frac{dN(t)}{dt} = KN(t)$$

$$\frac{dN(t)}{N(t)} = Kdt$$

$$\int \frac{dN(t)}{N(t)} = K\int dt$$

$$\ln[N(t)] = Kt + C$$

when $t = 0$; $N = N_0$

$$\log N_0 = C$$

$$\log[N(t)] = Kt + \log N_0$$

$$\log\frac{N(t)}{N_0} = Kt$$

$$\frac{N(t)}{N_0} = e^{Kt}$$

$$N(t) = N_0 e^{Kt}$$

when $t = T_1$ $N(T_1) = N_0 e^{KT_1}$

$$N_1 = N_0 e^{KT_1} \text{ --------(1)}$$

$t = T_2$ $N(T_2) = N_0 e^{KT_2}$

$$N_2 = N_0 e^{KT_2} \text{ --------(2)}$$

By (1); $\frac{N_1}{N_0} = e^{KT_1}$

$$KT_1 = \ln\left(\frac{N_1}{N_0}\right)$$

$$K = \frac{1}{T_1}\ln\left(\frac{N_1}{N_0}\right)$$

$$K = \ln\left(\frac{N_1}{N_0}\right)^{\frac{1}{T_1}}$$

Substituting K by (2);

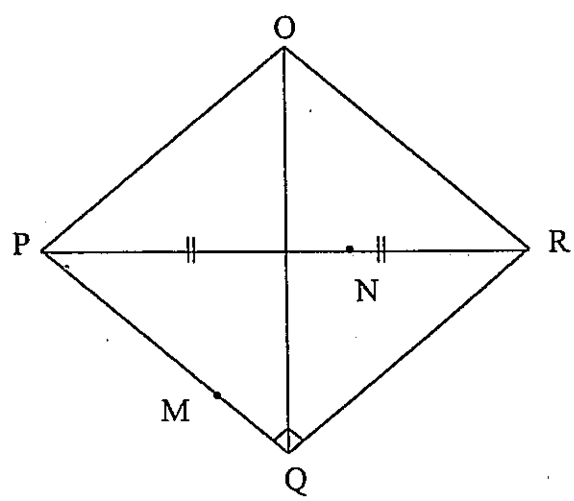
$$N_2 = N_0 e^{\left[\ln\left(\frac{N_1}{N_0}\right)^{\frac{1}{T_1}}\right]^{T_2}}$$

$$\frac{N_2}{N_0} = e^{\ln\left(\frac{N_1}{N_0}\right)^{\frac{T_2}{T_1}}}$$

$$\frac{N_2}{N_0} = \left(\frac{N_1}{N_0}\right)^{\frac{T_2}{T_1}}$$

$$\left(\frac{N_2}{N_0}\right)^{T_1} = \left(\frac{N_1}{N_0}\right)^{T_2}$$

(02)



Let M be the mid point of PQ. Join ON & OM, From PNOΔ & RNOΔ;

$\left.\begin{array}{l} PN = NR \\ PO = RO \end{array}\right\}$ [given data]

ON is common side for both PNOΔ & RNOΔ

$\therefore$ PNOΔ ≡ RNOΔ [PNOΔ & RNOΔ are congruent]

$P\hat{N}O + R\hat{N}O = \pi$

But $P\hat{N}O = R\hat{N}O$ [under congruent]

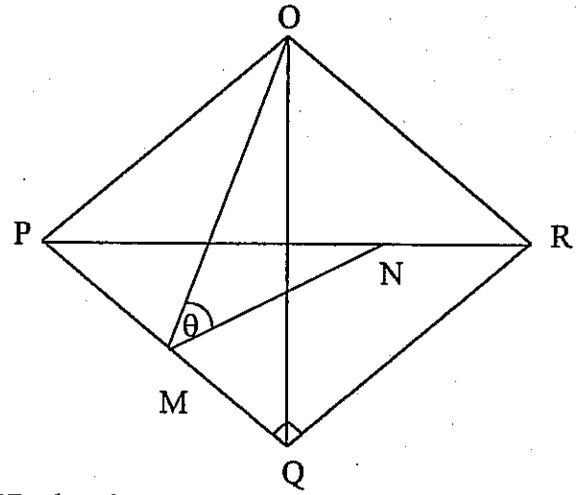
$\therefore 2P\hat{N}O = \pi$

$2P\hat{N}O = \pi/2$

$ON \perp PR$

$\therefore$ ON is perpendicular to the plane PQR.

(b)



Since $ON \perp PR$ the plane PQR

$$\cos O\hat{M}N = \frac{MN}{OM}$$

But $MN = \frac{1}{2}QR = \frac{a}{2}$

$$OM = \sqrt{OP^2 - PM^2}$$

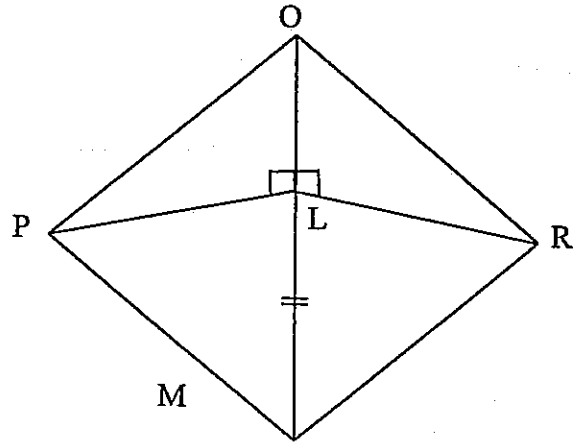
$$OM = \sqrt{a^2 - \frac{a^2}{4}}$$

$$OM = \frac{\sqrt{3}}{2}a$$

$$\therefore \cos O\hat{M}N = \frac{a/2}{\sqrt{3}a/2} = \frac{1}{\sqrt{3}}$$

$$O\hat{M}N = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.74^0$$

(iii)



The angle between planes OPQ & OQR is θ. Where $PL \perp OQ$ & $LR \perp OQ$ & L is the mid point of OQ ;

from PQR Triangle ; $PR^2 = PQ^2 + QR^2 = 2a^2$

$PR = \sqrt{2}a$

from PLO Triangle ; $PL^2 = PO^2 - LQ^2$

$PL^2 = a^2 - \left(\frac{a}{2}\right)^2 = \frac{3a^2}{4}$

$PL = \frac{\sqrt{3}}{2}a = LR$

from PLR Triangle ; $PR^2 = PL^2 + LR^2 - 2PL.LR\cos P\hat{L}R$

$$\left(\sqrt{2}a\right)^2 = \left(\frac{\sqrt{3}a}{2}\right)^2 + \left(\frac{\sqrt{3}a}{2}\right)^2 - 2\left(\frac{\sqrt{3}a}{2}\right)\left(\frac{\sqrt{3}a}{2}\right)\cos P\hat{L}R$$

$$\frac{3a^2}{2}\cos P\hat{L}R = \frac{6a^2}{4} - 2a^2$$

$$\frac{3a^2}{2}\cos P\hat{L}R = \frac{-2a^2}{4}$$

$$\cos P\hat{L}R = \frac{-2a^2}{4} \times \frac{2}{3a^2}$$

$$\cos P\hat{L}R = -\frac{1}{3}$$

$$P\hat{L}R = \cos^{-1}\left(-\frac{1}{3}\right)$$

angle between the planes OPQ & OQR is $\cos^{-1}(-1/3)$ $[109.47^0]$.

(03) (a) (i) $f(x) = x^3 - 5x + 1$

Let interval [2,3]

$f(2) = -1$

$f(3) = 13$

$f(2).f(3) < 0$

$\therefore$ There exists one root in [2,3].

$a_1 = 2,\ b_1 = 3$

hence $p_1 = \frac{2+3}{2} = 2.5$

$f(p_1) = 2.5^3 - 5(2.5) + 1$

$= 15.625 - 12.5 + 1$

$= 4.125 > 0$

The root is in between 2 & 2.5.

$\frac{b-a}{2^n} \le \varepsilon$

$\frac{3-2}{2^n} \le 10^{-4}$

$2^{-n} \le 10^{-4}$

$10^4 \le 2^n$

$\log_{10}\left(10^4\right) \le n\log_{10} 2$

$4 \le 0.301n$

$n \ge 13.289$

$n = 14$

n	a_n	b_n	p_n	$F(p_n)$
1	2	3	2.5	+4.125>0
2	2	2.5	2.25	+1.140625>0
3	2	2.25	2.125	-0.029296875<0
4	2.125	2.25	2.1875	+0.530029296>0
5	2.125	2.1875	2.15625	+0.244049072>0
6	2.125	2.15625	2.140625	+0.105808258>0
7	2.125	2.140625	2.1328125	+0.037865161>0
8	2.125	2.1328125	2.12890625	+0.00418668983>0
9	2.125	2.12890625	2.126953125	-0.012579433<0
10	2.126953125	2.12890625	2.127929688	-0.00420245566<0
11	2.127929688	2.12890625	2.128417969	-0.00000940527<0
12	2.128417969	2.12890625	2.12866211	+0.00208826593>0
13	2.128417969	2.12866211	2.12854004	+0.00103933948>0
14	2.128417969	2.12854004	2.128479005	+0.00051494763>0
15	2.128417969	2.128479005	2.128448487	+0.00025276523>0

$\therefore$ The required root = 2.1284

(b) (i) $x^3 - 3x + 4 = 0$

Let interval $[-2.2, -2.1]$

$x_0 = -2.2, \ x_1 = -2.1$

$f(x_0) = f(-2.2) = (-2.2)^3 - 3(-2.2) + 4 = -0.048$

$f(x_1) = f(-2.1) = (-2.1)^3 - 3(-2.1) + 4 = +1.039$

$f(x_0)f(x_1) < 0$

$\therefore$ The root lies between -2.2 & -2.1.

By the method of false position

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(-2.2)(1.039) - (-2.1)(-0.048)}{1.039 - (-0.048)}$$

$$x_2 = \frac{-2.2858 - 0.1008}{1.039 + 0.048}$$

$$x_2 = \frac{-2.3866}{1.087} = -2.195584177$$

$f(x_2) = f(-2.195584177) = 0.0027416705$

$f(x_0)f(x_2) < 0$

$\therefore$ The root lies between -2.2 & -2.195584177.

Then we can get $x_0 = -2.2$ & $x_1 = -2.195584177$ we can add the false position, until we get the answer in four decimal places.

Iterations	x_0	x_1	x_2	$f(x_2)$
1	-2.2	-2.1	-2.195584177	0.0027416705
2	-2.2	-2.195584177	-2.195822763	0.0000066778
3	-2.2	-2.195822763	-2.195823323	0.0000002574
4	-2.2	-2.195823323	-2.195823335	0.0000001198

∴ The required root = -2.1958.

(ii) $x^3 + x - 1 = 0$ near $x = 1$

Let interval $[0.6, 0.7]$

$x_0 = 0.6,\ x_1 = 0.7$

$f(x_0) = f(0.6) = (0.6)^3 + 0.6 - 1 = -0.184$

$f(x_1) = f(0.7) = (0.7)^3 + 0.7 - 1 = +0.043$

$f(x_0)f(x_2) < 0$

∴ The root lies between 0.6 & 0.7.

By the method of false position

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \frac{(0.6)(0.043) - (0.7)(-0.184)}{0.043 - (-0.184)}$$

$$x_2 = \frac{0.0258 + 0.1288}{0.227}$$

$$x_2 = \frac{0.1546}{0.227} = 0.681057268$$

$f(x_2) = f(0.681057268) = -0.003041808306$

$f(x_0)f(x_2) < 0$

∴ The root lies between 0.681057268 & 0.7.

Then we can get $x_0 = 0.681057268$ & $x_1 = 0.7$ we can add the false position, until we get the answer in four decimal places.

Iterations	x_0	x_1	x_2	$f(x_2)$
1	0.6	0.7	0.681057268	-0.003041808306
2	0.681057268	0.7	0.68230873	-0.0000457
3	0.68230873	0.7	0.6823275	0.000000728189
4	0.6823275	0.7	0.682327773	0.000000073031

The required root = 0.6823

(c) $f(x) = x^2 - 4x + 3.9999$

$f'(x) = 2x - 4$

(a) given $x_0 = 2.1$

Newton raphson method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

$$p_1 = 2.1 - \frac{f(2.1)}{f'(2.1)}$$

n	p_{n-1}	p_n	$f(p_{n-1})$	$f'(p_{n-1})$
1	2.1	2.0505	0.0099	0.2
2	2.0505	2.026240099	0.00245025	0.101
3	2.026240099	2.01502553	0.00058854279	0.052480198
4	2.01502553	2.01084042	0.00012576655	0.03005106
5	2.01084042	2.010032564	0.0000175147	0.02168084
6	2.010032564	2.01000007	0.00000065234	0.020065128

∴ root of x = 2.01

∴ There is quadratic convergence.

(b) $x_0 = 1.9$

Newton raphson method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

$$p_1 = 1.9 - \frac{f(1.9)}{f'(1.9)}$$

n	p_{n-1}	p_n	$f(p_{n-1})$	$f'(p_{n-1})$
1	1.9	1.9495	0.0099	-0.2
2	1.9495	1.9737599	0.00245025	-0.101
3	1.9737599	1.98497448	0.000588543	-0.0524802
4	1.98497448	1.989159532	0.000125766	-0.03005104
5	1.989159532	1.989967385	0.00001751546	-0.021680936
6	1.989967385	1.989999979	0.00000065336	-0.02006523

∴ root of x = 1.99

∴ There is quadratic convergence.

(c) $x_0 = 2.0$

Newton raphson method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

$$p_1 = 2.0 - \frac{f(2.0)}{f'(2.0)}$$

$$f(2.0) = 4 - 8 + 3.9999 = -0.0001$$

$$f(2.0) = 0$$

$$p_1 = 2.0 - \frac{0.0001}{0}$$

Therefore can't determine root. There is no quadratic convergence.

(d) $x_0 = 2.02$

Newton raphson method

$$p_n = p_{n-1} - \frac{f(p_{n-1})}{f'(p_{n-1})}$$

$$p_1 = 2.02 - \frac{f(2.02)}{f'(2.02)}$$

n	p_{n-1}	p_n	$f(p_{n-1})$	$f'(p_{n-1})$
1	2.02	2.0125	0.0003	0.04
2	2.0125	2.01025	0.00005625	0.025
3	2.01025	2.010003073	0.0000050625	0.0205
4	2.010003073	2.010000024	0.00000006146	0.020006146

There is no different root. If we assume any value near 2.01 our root is 2.01.

(04) (a) (i) **Jacoi Algorithm**

Consider the following system of equations.

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \ldots\ldots\ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots\ldots\ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1 + a_{n2}x_2 + \ldots\ldots\ldots + a_{nn}x_n = b_n \end{array}\right\} \text{\_\_\_\_\_\_(A)}$$

Let x_i be the general terms;

Then the iterative formulae for the jacobi method for the system of equation in (A) can be given as follows.

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - a_{i1}x_1^{(k)} \ldots\ldots\ldots - a_{i,i-1}x_{i-1}^{(k)} - \ldots\ldots\ldots - a_{in}x_n^{(k)}\right]$$

where $i = 1, 2, \ldots\ldots\ldots n$.

It could be expressed as $a_{ii}x_i^{(k+1)} = b_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij}x_j^{(k)}$ in matrix form.

<u>Gauss – Seidal Algorithm</u>

In the J method, values of the k^{th} cycle are used to get values of the $(k+1)^{th}$ cycle. But in this method, the latest available values are taken. That as to get $x_i^{(k+1)}$, we use $x_1^{(k+1)}$, $x_2^{(k+1)}$, ... $x_{i-1}^{(k+1)}$ and $x_i^{(k)}$, $x_{i+1}^{(k)}$, ... $x_n^{(k)}$ values. This is the basis of the G. S. method. Therefore the iterative formula, for the general case can be written as follows. Its matrix form is

$$x_i^{(k+1)} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{N} a_{ij}x_j^{(k)}\right]$$

$i = 2, \ldots\ldots\ldots\ldots, n$

$k = 2, 3, \ldots\ldots\ldots\ldots$

(ii) (a) $5x + 2y + z = 12 \Rightarrow x = \frac{1}{5}(12 - 2y - z)$

$x + 4y + 2z = 15 \Rightarrow y = \frac{1}{4}(15 - x - 2z)$

$x + 2y + 5z = 20 \Rightarrow z = \frac{1}{5}(20 - x - 2y)$

Let $x^{(0)} = 0,\ y^{(0)} = 0,\ z^{(0)} = 0$

$x^{(1)} = \frac{1}{5}(12 - 0 - 0) = \frac{12}{5} = 2.4$

$y^{(1)} = \frac{1}{4}(15 - 0 - 0) = \frac{15}{4} = 3.75$

$z^{(1)} = \frac{1}{5}(20 - 0 - 0) = 4.0$

$x^{(2)} = \frac{1}{5}(12 - 7.5 - 4) = 0.1$

$y^{(2)} = \frac{1}{4}(15 - 2.4 - 8) = 1.15$

$z^{(2)} = \frac{1}{5}(20 - 2.4 - 7.5) = 2.02$

Iterations	x	y	z
0	0	0	0
1	2.4	3.75	4.0
2	0.1	1.15	2.02
3	1.536	2.715	3.52
4	0.61	1.606	2.6068
5	1.23624	2.2941	3.2356
6	0.83524	1.82314	2.835112

7	1.1037216	2.123634	3.103696
8	0.9298072	1.9222216	2.92980208
9	1.045150944	2.05264716	3.04514992
10	0.969911152	1.966137304	2.969910947
11	1.019562889	2.022566739	3.019562848
12	0.987060734	1.985327854	2.987060727
13	1.008456713	2.009704454	3.008456712
14	0.994426876	1.993657466	2.994426876
15	1.003651638	2.004179843	3.003651638
16	0.997597735	1.997261272	2.997597735
17	1.001575944	2.001801699	3.001575944

$|x_{17} - x_{16}| = 0.00398 < 0.005$

$|y_{17} - y_{16}| = 0.00454 < 0.005$

$|z_{17} - z_{16}| = 0.00398 < 0.005$

Therefore

$x = 1.001575944 \;\; x = 1$

$y = 2.001801699 \;\; y = 2$

$z = 3.001575944 \;\; z = 3$

(b) **<u>Gauss – Seidal Method</u>**

$$5x + 2y + z = 12 \Rightarrow x = \frac{1}{5}(12 - 2y - z)$$

$$x + 4y + 2z = 15 \Rightarrow y = \frac{1}{4}(15 - x - 2z)$$

$$x + 2y + 5z = 20 \Rightarrow z = \frac{1}{5}(20 - x - 2y)$$

Let $y^{(0)} = 0,\ z^{(0)} = 0$

$$x^{(1)} = \frac{1}{5}(12) = 2.4$$

$$y^{(1)} = \frac{1}{4}(15 - 2.4) = 3.15$$

$$z^{(1)} = \frac{1}{5}(20 - 2.4 - 2 \times 3.15) = 2.26$$

Iterations	x	y	z
1	2.4	3.15	2.26
2	0.688	2.448	2.8832
3	0.84416	2.09736	2.992224
4	0.9626112	2.0132352	3.00218368
5	0.994269184	2.000340864	3.001009818

6	0.99966169	1.999579669	3.000235794
7	1.000120974	1.99985186	3.000035061

$|x_7 - x_6| = 0.000459 < 0.05$

$|y_7 - y_6| = 0.000272 < 0.05$

$|z_7 - z_6| = 0.0002007 < 0.05$

Therefore

$x = 1.000120974 \quad x = 1$

$y = 1.99985186 \quad y = 2$

$z = 3.000035061 \quad z = 3$

Gauss – Seidal method is most suitable. Because this method converges faster for the problem with less iterations than Jacobi's method.

(b) $10x - 2y - z - w = 3 \Rightarrow x = \frac{1}{10}(3 + 2y + z + w)$

$-2x + 10y - z - w = 15 \Rightarrow y = \frac{1}{10}(2x + z + w + 15)$

$-x - y + 10z - 2w = 27 \Rightarrow z = \frac{1}{10}(27 + x + y + 2w)$

$-x - y - 2z + 10w = -9 \Rightarrow w = \frac{1}{10}(-9 + x + y + 2z)$

Let $y^{(0)} = 0,\ z^{(0)} = 0,\ w^{(0)} = 0$

$x^{(1)} = \frac{3}{10} = 0.3$

$y^{(1)} = \frac{1}{10}(2 \times 0.3 + 15) = 1.56$

$z^{(1)} = \frac{1}{10}(27 + 0.3 + 1.56) = 2.886$

$w^{(1)} = \frac{1}{10}(-9 + 0.3 + 1.56 + 5.772) = -0.1368$

Iterations	x	y	z	w
1	0.3	1.56	2.886	-0.1368
2	0.88692	1.952304	2.9565621	-0.02476518
3	0.9836405	1.989907792	2.992401793	-0.0041648122
4	0.996805267	1.998184752	2.99866604	-0.0007677901
5	0.999426775	1.99967518	2.999756638	-0.0001384769

$|x^{(4)} - x^{(5)}| = 0.0026215 < 0.005$

$\left|y^{(4)} - y^{(5)}\right| = 0.00149043 < 0.005$

$\left|z^{(4)} - z^{(5)}\right| = 0.0006293 < 0.005$

Therefore

$x = 0.999426775 \quad x = 1$

$y = 1.99967518 \quad y = 2$

$z = 2.999756638 \quad z = 3$

$w = -0.0001384769 \quad w = 0$

(05) (a) $10x - 7y - 3z + 5u = 6$ __________(1)

$-6x + 8y - z - 4u = 5$ __________(2)

$5x - 2y + 9z + 4u = 7$ __________(3)

$3x + y + 4z + 11u = 2$ __________(4)

$(1)/10 \Rightarrow x - 0.7y - 0.3z + 0.5u = 0.6$ __________(1A)

$(2) + (1A) \times 6 \Rightarrow 3.8y - 2.8z - u = 8.6$ __________(2A)

$(3) - (1A) \times 5 \Rightarrow 1.5y + 10.5z + 1.5u = 4$ __________(3A)

$(4) - (1A) \times 3 \Rightarrow 3.1y + 4.9z + 9.5u = 0.2$ __________(4A)

Dividing Equation (2A) by the 2nd pivot 3.8 ;

$(2A)/3.8 \Rightarrow y - 0.7368z - 0.2632u = 2.2632$ __________(2B)

$(3A) - (2B) \times 1.5 \Rightarrow 11.6052z + 1.8948u = 0.6052$ __________(3B)

$(4A) - (2B) \times 3.1 \Rightarrow 7.18408z + 10.3159u = -6.8159$ __________(4B)

Dividing Equation (3B) by the 3rd pivot 11.6052 ;

$\frac{[3B]}{11.6052} \Rightarrow z + 0.1633u = 0.0521$ __________(3C)

$(4B) - 7.18408 \times (3C)$

$[10.3159 - 1.1732]u = -6.8159 - 0.3743$

$9.1427u = -7.1902$ __________(4C)

$(4C) \Rightarrow u = -0.7864$

$(3C) \Rightarrow z = 0.0521 - 0.1633(-0.7864)$

$z = 0.1805$

$(2B) \Rightarrow y = 2.2632 + 0.13299 - 0.20698$

$y = 2.1892$

$(1A) \Rightarrow x = 0.6 + 0.7(2.1892) + 0.3(0.1805) - 0.5(-0.7864)$

$x = 0.6 + 1.53244 + 0.5415 + 0.3932$

$x = 2.5798$

$\therefore$ The solutions are ;

$$\begin{Bmatrix} x = 2.5798 \\ y = 2.1892 \\ z = 0.1805 \\ u = -0.7864 \end{Bmatrix}$$

(b) Let $A = \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix}$

$A = IA$

$$\begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\begin{bmatrix} 1 & 1/4 & 1/2 \\ 2 & 3 & -1 \\ 1 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \qquad \left(R_1 \to \frac{1}{4}R_1\right)$$

$$\begin{bmatrix} 1 & 1/4 & 1/2 \\ 0 & 5/2 & -2 \\ 0 & -9/4 & 3/2 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 \\ -1/2 & 1 & 0 \\ -1/4 & 0 & 1 \end{bmatrix} A \qquad \begin{matrix} (R_2 \to R_2 - 2R_1) \\ (R_3 \to R_3 - R_1) \end{matrix}$$

$$\begin{bmatrix} 1 & 1/4 & 1/2 \\ 0 & 1 & -4/5 \\ 0 & -9/4 & 3/2 \end{bmatrix} = \begin{bmatrix} 1/4 & 0 & 0 \\ -1/5 & 2/5 & 0 \\ -1/4 & 0 & 1 \end{bmatrix} A \qquad \left(R_2 \to \frac{2}{5}R_2\right)$$

$$\begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & -4/5 \\ 0 & 0 & -3/10 \end{bmatrix} = \begin{bmatrix} 2/9 & 0 & 1/9 \\ -1/5 & 2/5 & 0 \\ -7/10 & 9/10 & 1 \end{bmatrix} A \qquad \begin{matrix} \left(R_1 \to R_1 + \frac{1}{9}R_3\right) \\ \left(R_3 \to R_3 + \frac{9}{4}R_2\right) \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 2/3 \\ 0 & 1 & -4/5 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2/9 & 0 & 1/9 \\ -1/5 & 2/5 & 0 \\ 7/3 & -3 & -10/3 \end{bmatrix} A \quad \left(R_3 \to \frac{-10}{3}R_3\right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -4/3 & 2 & 7/3 \\ 5/3 & -2 & -8/3 \\ 7/3 & -3 & -10/3 \end{bmatrix} A \qquad \begin{matrix} \left(R_1 \to R_1 - \frac{2}{3}R_3\right) \\ \left(R_2 \to R_2 + \frac{4}{5}R_3\right) \end{matrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{4}{3} & 2 & \frac{7}{3} \\ \frac{5}{3} & -2 & -\frac{8}{3} \\ \frac{7}{3} & -3 & -\frac{10}{3} \end{bmatrix}$$

(c) $2x - y + z = 5$

$2y - z + u = 1$

$x + 2z - u = 8$

$x + y + 2u = 2$

$$\underbrace{\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 0 & 2 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 5 \\ 1 \\ 8 \\ 2 \end{bmatrix}}_{\underline{B}}$$

Let $LU = A$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 \end{bmatrix}}_{\underline{A}} \underbrace{\begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 2 & -1 \\ 1 & 1 & 0 & 2 \end{bmatrix}}_{\underline{B}}$$

$u_{11} = 2$	$u_{12} = -1$	$u_{13} = 1$	$u_{14} = 0$
$\ell_{21}u_{11} = 0$	$\ell_{21}u_{12} + u_{22} = 2$	$\ell_{21}u_{13} + u_{23} = -1$	$\ell_{21}u_{14} + u_{24} = 1$
$\ell_{21} = 0$	$u_{22} = 2$	$u_{23} = -1$	$u_{24} = 1$

$\ell_{31}u_{11} = 1$ $\qquad \ell_{31}u_{12} + \ell_{32}u_{22} = 0$

$\ell_{31} = \frac{1}{2}$ $\qquad \frac{1}{2} \times -1 + \ell_{32} \times 2 = 0$

$\ell_{32} = \frac{1}{4}$

$$\ell_{31}u_{13} + \ell_{32}u_{23} + u_{33} = \frac{1}{2} \times 1 + \frac{1}{4} \times -1 + u_{33} = 2$$

$$u_{33} = \frac{7}{4}$$

$$\ell_{31}u_{14} + \ell_{32}u_{24} + u_{34} = \frac{1}{2} \times 0 + \frac{1}{4} \times 1 + u_{34} = -1$$

$$u_{34} = -1 - \frac{1}{4} = -\frac{5}{4}$$

$\ell_{41}u_{11} = \ell_{41} \times 2 = 1$ $\qquad \ell_{41}u_{12} + \ell_{42}u_{22} = 1$

$$\ell_{41} = \tfrac{1}{2} \qquad \tfrac{1}{2}\times -1 + \ell_{42}\times 2 = 1$$

$$\ell_{42} = \frac{1+1/2}{2} = \frac{3}{4}$$

$$\ell_{41}u_{13} + \ell_{42}u_{23} + \ell_{43}u_{33} = 0$$

$$\tfrac{1}{2}\times 1 + \tfrac{3}{4}\times(-1) + \ell_{43}\left(\tfrac{7}{4}\right) = 0$$

$$\tfrac{7}{4}\times \ell_{43} = \tfrac{3}{4} - \tfrac{1}{2} = \frac{3-2}{4}$$

$$\ell_{43} = \tfrac{1}{7}$$

$$\ell_{41}u_{14} + \ell_{42}u_{24} + \ell_{43}u_{34} + u_{44} = 2$$

$$\tfrac{1}{2}\times 0 + \tfrac{3}{4}\times 1 + \tfrac{1}{7}\times -\tfrac{5}{4} + u_{44} = 2$$

$$u_{44} = 2 + \tfrac{5}{28} - \frac{3}{4}$$

$$u_{44} = \frac{56+5-21}{28}$$

$$u_{44} = \frac{10}{7}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \tfrac{1}{2} & \tfrac{1}{4} & 1 & 0 \\ \tfrac{1}{2} & \tfrac{3}{4} & \tfrac{1}{7} & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & \tfrac{7}{4} & -\tfrac{5}{4} \\ 0 & 0 & 0 & \tfrac{10}{7} \end{bmatrix}$$

$$Ly = B$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \tfrac{1}{2} & \tfrac{1}{4} & 1 & 0 \\ \tfrac{1}{2} & \tfrac{3}{4} & \tfrac{1}{7} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 8 \\ 2 \end{bmatrix}$$

Equating each term to the corresponding term on the right ;

$$y_1 = 5, \qquad y_2 = 1$$

$$\tfrac{1}{2}y_1 + \tfrac{1}{4}y_2 + y_3 = 8$$

$$y_3 = 8 - \frac{1}{2}\times 5 - \frac{1}{4}\times 1 = 8 - \frac{5}{2} - \frac{1}{4} = \frac{32-10-1}{4} = \frac{21}{4}$$

$$\frac{1}{2}y_1 + \frac{3}{4}y_2 + \frac{1}{7}y_3 + y_4 = 2$$

$$y_4 = 2 - \frac{1}{2}\times 5 - \frac{3}{4}\times 1 - \frac{1}{7}\times\frac{21}{4}$$
$$= 2 - \frac{5}{2} - \frac{3}{4} - \frac{3}{4}$$
$$= -2$$

$$y = \begin{bmatrix} 5 \\ 1 \\ 21/4 \\ -2 \end{bmatrix}$$

$$Ux = y$$

$$\begin{bmatrix} 2 & -1 & 1 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & 0 & 7/4 & -5/4 \\ 0 & 0 & 0 & 10/7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ u \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 21/4 \\ -2 \end{bmatrix}$$

Equating each term to the corresponding term on the right ;

$$10/7\, u = -2$$
$$u = -7/5$$

$$7/4\, z - 5/4\, u = 21/4$$
$$z = 2$$

$$2y - z + u = 1$$
$$y = 11/5$$

$$2x - y + z = 5$$
$$x = 13/5$$

∴ Solutions are ;

$$x = 13/5, \quad y = 11/5, \quad z = 2, \quad u = -7/5$$