THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY– FOUNDATION LEVEL 02

FINAL EXAMINATION - 2007

MPZ 2230 – MATHEMATICS – PAPER II

DURATION – THREE (03) HOURS

DATE : 04th May 2008 TIME: 9.30 a.m. – 12.30 p.m.

ANSWER SIX (06) QUESTIONS ONLY BY SELECTING AT LEAST ONE QUESTION FROM EACH SECTION. YOU CAN USE CALCULATORS. YOU CANT USE MOBILE PHONES AS CALCULATORS.

SECTION – A

01. Let H be the point in the plane of the triangle ABC such that BH is perpendicular to AC and CH is perpendicular to AB. With respect to a set of rectangular Cartesian axes in the plane of the triangle. $A \equiv (4,3)$. The equations of BH and CH are $x + y + 1 = 0$ and $5x + 3y - 5 = 0$ respectively.

    Find the equations of the sides AB and AC. Find the coordinates of the points B, C and H. Verify that AH and BC are perpendicular. Find the area of the triangle ABC.

    Through each vertex of the triangle ABC a straight line paralled to the opposite side is drawn and these three lines form the triangle $A'B'C'$. by using geometrical methods (without finding the coordinates of the points). Show that H is the equidistant from $A'B'$ and C'.

    Deduce that the area of the triangle $A'B'C'$.

02. S is the circle passing through the points $A \equiv (8,0)$, $B \equiv (0,6)$ and $C \equiv (7,7)$. Find the equation of S. Show that S passes through the point P (8,6). If the tangents at B and P meet at the point Q. Find the coordinates of Q; Prove that $PQ = \frac{20}{3}$.

    Find the locus of the centres of the circles which touch the circle S externally and also the straight line $x + 3 = 0$.

03. Find the equation of the normal to the parabola $y^2 = 4ax$ at the point P $(at^2,2at)$. The normal at the point P meets the x axis at G and the mid point of PG is N.

i. Find the coordinates of the points G and N.
ii. Find the locus of the point N as P varies on the parabola.
iii. Given that $S \equiv (a,0)$, then show that PG is perpendicular to SN.
iv. If the triangle SPG is equilateral find the coordinates of the point P.

04. Let $P_1 \equiv (a\cos\alpha, b\sin\alpha)$ and $P_2 \equiv (a\cos\beta, b\sin\beta)$ be two distinct points on the ellipse $S \equiv \frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$. Show that the equation of the chord P_1P_2 is $\frac{x}{a}\cos(\frac{\alpha+\beta}{2})+\frac{y}{b}\sin(\frac{\alpha+\beta}{2})=\cos(\frac{\alpha-\beta}{2})$ Write down the equation of the tangent to the ellipse s=0 at the point $P \equiv (a\cos\theta, b\sin\theta)$ Let N be the foot of the perpendicular from the centre of the ellipse S = 0 on the tangent at P. Find the coordinate of the mid point of ON. Hence show that the locus of the mid point of ON as P varies on the ellipse is $4(x^2+y^2)^2 = a^2x^2+b^2y^2$.

05. Prove that if $16p^2 - 9q^2=1$, then the straight line $px + qy = 1$, touches the hyperbola $S \equiv \frac{x^2}{4^2}-\frac{y^2}{3^2}-1=0$. The asymptotes of the hyperbola, meet the circle $x^2 + y^2 - 2ky - 25 = 0$ at the points $P_1 \equiv (x_1, y_1)$ and $P_2 \equiv (x_2, y_2)$. Where k is a constant. Given that P_1 is in the 1st quadrant and P_2 is in the 4th quadrant.

Show that $\frac{y_1}{3}=\lambda+\sqrt{1+\lambda^2}$ and $\frac{x_2}{4}=-\lambda+\sqrt{1+\lambda^2}$

Where $\lambda=\frac{3k}{25}$;

Hence show that the line P_1P_2 touches the hyperbola S = 0 for all values of k.

06. (a). $f(\theta)=\frac{1}{4\sin\theta-3\cos\theta+6}$, for $\theta\in\Re$

i. Show that $\frac{1}{11}\le f(\theta)\le 1$ for all $\theta\in\Re$

ii. Solve the equation $f(\theta)=1$ giving all the solutions between 0° and 360^0.

(b) Show that if the equation $\tan^{-1}(x + a) - \tan^{-1}(x+b) = \frac{\pi}{4}$ (where a and b are constants) has real solutions for x, then $(a-b+2)^2 \geq 8$. Hence solve the equation $\tan^{-1}(x+1)-\tan^{-1}(x-1) = \frac{\pi}{4}$.

(c) By using the sine rule for a triangle, with usual notation show that if $a = c + \lambda b$ where $\lambda \in \Re$, then $\lambda Cos\frac{B}{2} = Cos\left(C + \frac{B}{2}\right)$.

SECTION – B

07. State De Moivre's Theorem,

By assuming $(Cos\ \theta + j\ Sin\theta)^n = Cos\ n\theta + jSin\ n\theta$ for $n \in Z^+$; Prove the result for $n \in Z^-$.

i. Show that for positive integral n,

$$(Sin\ \theta + j\ Cos\ \theta)^n = Cos\ n\left(\frac{\pi}{2} - \theta\right) + j\ Sin\ n\left(\frac{\pi}{2} - \theta\right).$$

Deduce the values $(1+j)^8$ and $(1-j)^8$.

ii. Show that $\left\{\frac{1 + Sin\theta + jCos\theta}{1 + Sin\theta - jCos\theta}\right\}^8 = Cos(4\pi - 8\theta) + jSin(4\pi - 8\theta)$

Deduce that $\left[\frac{1+j}{1-j}\right]^8 = 1$

08. (a) Using corresponding loci in the Argand diagram depict the points representing the complex numbers z satisfying both equations

$Arg\ (z-1) - Arg\ (z-i) = \frac{\pi}{4}$ and $|z| = 1$; Where Arg refers to the principal value lying between $-\pi$ and π.

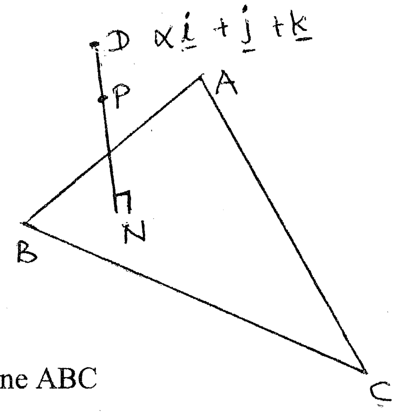
(b) The position vectors of the points A, B and C are respectively

$\overrightarrow{OA} = \sqrt{3}\,\mathbf{i}+\mathbf{j}$

$\overrightarrow{OB} = \sqrt{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$ and $\overrightarrow{OC} = -\mathbf{i}+\sqrt{3}\,\mathbf{j}$

Find the magnitudes of the vectors $\overrightarrow{OA}, \overrightarrow{OB}$ and $\overrightarrow{OC}$. Hence show that the points A, B, C are on a circle, and find its centre and the radius. Find the position vector of the point D, such that ABDC is a parallelogram.

09. Let the position vectors of the points A, B and C with respect to an origin O be $3\mathbf{i}+3\mathbf{j} - \mathbf{k}$, $4\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ and $6\mathbf{i}+10\mathbf{j}-7\mathbf{k}$ respectively.



i. Find $\overrightarrow{BA}$ and $\overrightarrow{AC}$

ii. Find $\overrightarrow{BA} \cdot \overrightarrow{AC}$

iii. Find the angle $B\hat{A}C$

iv. Find $\overrightarrow{BA} \times \overrightarrow{AC}$

v. Find a unit vector perpendicular to the plane ABC

vi. Find the perpendicular distance from C to the line AB

vii. DN is the line such that DN is perpendicular to the plane ABC and $\overrightarrow{OD} = \alpha\mathbf{i} + \mathbf{j} + \mathbf{k}$. Let $\overrightarrow{OP} = x\mathbf{i} + y\mathbf{j} + z\,\mathbf{k}$. Find the expression for $\overrightarrow{DP}.(\overrightarrow{BA} \times \overrightarrow{AC})$ and hence show that $\overrightarrow{OP}.(\overrightarrow{BA} \times \overrightarrow{AC}) = \overrightarrow{OD}.\overrightarrow{BA} \times \overrightarrow{AC}$ and $2x+z=2\alpha+1$.

SECTION - C

10 i. A husband and wife appear in an interview for two vacancies in the same post. The probability of husband's selection is 1/7 and that of wife's selection is 2/7. What is the probability that

a) only one of them will be selected
b) both of them will be selected.
c) non of them will be selected
d) At least one of them will be selected.

ii. A regular octahedron which has its triangular faces numbered from 1 to 8 is to be used as a die. The score for a throw is the number on the face that comes uppermost when the die is thrown on a horizontal table. If two such dice are thrown, find the probability that

a) the total score on the two dice is at least 14

b) the difference between the scores is 02.

Two players A and B throw both dice in turns (with A starting), and the first player to obtain a difference of 2 between the scores on the dice wins the game.

Find the probability that

c) A wins at his first throw

d) B wins at his first throw

e) A wins at his third throw

11. i. The following table givens a grouped frequency distribution of the lifetimes of a random sample for 1600 electric bulbs taken from the out put of a particular factory.

Lifetimes in Hours	No of Bulbs
1000 – 1500	120
1500 – 2000	220
2000 – 2500	280
2500 – 3000	420
3000 – 3500	300
3500 – 4000	180
4000 – 4500	80

Estimate the mode, median, mean and standard deviation of the lifetime.

ii. The mean and variance of 05 observations are 4.4 and 6.64 respectively. If three of the five observations are 2,1 and 6. Find the other two observations.

12. Please answer any two parts from (a), (b) and (c).

(a) A particle P projected horizontally with speed u from the lowest point A of the smooth inside surface of a fixed hollow sphere of internal radius a and centre O. Find the expression for velocity V and normal reaction R of the particle a the point Q where $A\hat{O}Q = \theta$.

i. In the case when $u^2=ga$ show that P does not leave the surface of the sphere. Show also that when P has moved halfway along its path from A towards the point at which it first comes to rest, its speed is $\sqrt{ga(\sqrt{3}-1)}$.

ii. Find u^2 in terms of g and a in the case when P leaves the surface at a height $\frac{3a}{2}$ above A, and in terms of a and g, the speed of P as it leaves the surface.

(b) One end of a light elastic string of natural length 2a and modulus 4mg is attached to a fixed point 'A' of a smooth horizontal table. A particle of mass m is attached to the other end of the string. The particle is released from rest from a point 'C' of the table where AC=3a. Show that the particles reaches the point 'C' again after a time of $(4+\pi)\sqrt{2a/g}$.

(c) A string passes round the rim of a wheel of radius a fixed with its axis horizontal and carries masses m, and m_2 at its ends. Assuming that the friction at the axis of the wheel is negligible, show that, it $m_2 > m_1$ the acceleration of the masses is $\dfrac{(m_2 - m_1)g}{m_2 + m_1 + \dfrac{I}{a^2}}$. Where I is the moment of inertia of the wheel about its axis.