THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY- LEVEL 03
FINAL EXAMINATION – 2008/2009
MPZ 3230 – ENGINEERING MATHEMATICS I
DURATION – THREE (03) HOURS

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DATE : 13th March 2009 **TIME: 9.30 a.m. – 12.30 p.m.**

Answer only six (06) questions selecting at least two from sections A and B.

Instructions:

- **State any assumption you use.**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**

SECTION - A

01. i. Define dot and cross product of two vectors $\underline{a}$ and $\underline{b}$.

ii. The vertices of a triangle are

A = (5, 0, -6)
B = (1, 1, 3) and
C = (-1, -2, -3)

a) Show that the triangle ABC is isosceles.

b) Show that the ABC is right angled triangle.

c) Find the area of the triangle.

iii. a) Write a vector that is perpendicular to both vectors $\underline{a} = \underline{i} - 2\underline{j} + 3\underline{k}$ and $\underline{b} = 4\underline{j} - 5\underline{k}$

b) Show that the vectors $\underline{a}, \underline{b}, \underline{c}$ are Coplanar.
$\underline{a} = 3\underline{i} - 2\underline{j} + \underline{k}$, $\underline{b} = 5\underline{i} + 4\underline{j} - 3\underline{k}$, $\underline{c} = 11\underline{i} - \underline{k}$

02. A sphere of radius 3 units is moving so that its centre, initially at the origin, has a constant speed of 5 units along OX. Another sphere of radius 2 units has a constant velocity vector $4\underset{\sim}{i} - \underset{\sim}{j}$ and its centre is initially at the point with position vector $4\underset{\sim}{i} + 11\underset{\sim}{j}$. ($\underset{\sim}{i}$ and $\underset{\sim}{j}$ are unit vectors paralled to OX and OY respectively)

(i) Find the position vector of the two spheres.

(ii) Determine the time for the collision of two spheres.

(iii) Find the position vectors of the centers of both spheres at that time.

03. (i) Show that

$$A = \begin{pmatrix} Cos\,\theta & 0 & Sin\,\theta \\ 0 & 1 & 0 \\ -Sin\,\theta & 0 & Cos\,\theta \end{pmatrix}$$

is an orthogonal matrix.

(ii) For what values of k does the system

$$kx + y + z = 1$$

$$x + ky + z = 1$$

$$x + y + kz = 1$$

Have (a) no solution

(b) a unique solution and

(c) infinitely many solutions.

04. (i) By using the Gauss elimination method (in matrix form) Solve.

$$-x_1 + x_2 + 2x_3 = 2$$
$$3x_1 - x_2 + x_3 = 6$$
$$-x_1 + 3x_2 + 4x_3 = 4$$

(ii) Without expanding the determinant, prove the following.

(a) $$|A| = \begin{vmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix}$$

$$= (a+3b)(a-b)^3$$

(b) $$|A| = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$$
$$= (b-c)(c-a)(a-b)(a+b+c)$$

05. (i) The probability that a trainee will remain with a company is 0.6. The probability that an employee earns more than Rs. 10,000 per year is 0.5. The probability that an employee is a trainee who remained with the company or who earns more than Rs. 10,000 per year is 0.7. What is the probability that an employee earns more than Rs. 10,000 per year given that he is a trainee who stayed with the company.

(ii) Suppose there is a chance for a newly constructed building to collapse whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the building collapses is 95%, if the design is faulty, and otherwise it is 45%. It is seen that the building collapsed, what is the probability that it is due to faulty design?

06. (i) (a) In a children's game the dice have two faces marked with a 1, two faces marked with a 2, and two faces with a 3. Write down the probability distribution for the total of the numbers shown on two such dice.

(b) What is the expected value and variance of that probability distribution.

(ii) A continuous random variable X has pdf

$$f(x) = \frac{3}{4}(1+x^2) \qquad 0 \le x \le 1$$

If $E(x) = \mu$ & $V(x) = \sigma^2$
Find $P\{|x - \mu| < \sigma\}$

SECTION – B

07. (i) Find from the following table the area bounded by the curve and the x – axis from x = 7 . 47 to 7.52.

(a) Using trapezoidal rule and

(b) Simpson's rule

x	7.47	7.48	7.49	7.50	7.51	7.52
F(x)	1.93	1.95	1.98	2.01	2.03	2.06

(ii) The expectation of life at different ages of male is shown below.

Age (year)	20	25	30	35	40
Expectation of life	33	29.8	26.6	23.5	20.5

Use Newton's formula to estimate the expectation of life at the age of 32.

(iii) Use Lagrange's interpolation formula to find Y, when X = 0, given the following table.

X	-1	-2	2	4
Y	-1	-9	11	69

08. (i) Obtained the Laplace transform of

(a) $t^3 + t^2 + 5$

(b) t Sin wt, where w is a constant.

(ii) Solve the following boundary value problem, using the Laplace transform method.

$$\frac{d^2y}{dx^2} - 4y = e^{-3x}\sin 2x$$

Subject to the boundary conditions $y(0) = y'(0) = 0$

09. (i) Neutrons in an atomic pile increase at the rate proportional to the number of neutrons present at any instant. If no Neutrons are initially present and N_1 and N_2 Neutrons are present at time T_1 and T_2 respectively.

Show that, $$\left(\frac{N_2}{N_0}\right)^{T_1} = \left(\frac{N_1}{N_0}\right)^{T_2}$$

N(t) is number of neutrons at any time t.

(ii) The equation of motion of a body performing damped forced vibrations is given by;

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = Cost$$

(a) Find the complementary function.

(b) Solve the differential equation completely, given that x = 0.1 and $\frac{dx}{dt} = 0$ when t = 0.

(c) Show that the "steady state" can be written in the form K sin (t + α), giving the numerical values of K and α.

10. (i) Solve the following ordinary differential equations.

(a) $y(xy+1)+x(1+xy+x^2y^2)\frac{dy}{dx}=0$

(b) $\frac{dy}{dx}=\frac{y^2+xy^2}{x^2y-x^2}$

(c) $(1+x^2)\frac{dy}{dx}+3xy=5x$, given that $y=2$ when $x=1$

(ii) Solve the following initial value problem.

$$\frac{d^2x}{dt^2}+4\frac{dx}{dt}+3x=e^{-3t}$$

Given that at $t=0$, $x=½$ and $\frac{dx}{dt}=-2$

11. (i) Simplify the following functions using Karnaugh's map.

F(A, B, C, D) $=\sum$ 5,6,8,10,12,15

(ii) OA, OB, OC are 3 mutually perpendicular lines. Their length are a, b and c respectively. ON is the perpendicular drawn from O to the plane ABC, when CN produce it meet AB at point D.

(Hint : If a line l is perpendicular to each of two intersecting lines a and b, then l is perpendicular to any line C on plane made by lines a and b.)

Using this result prove the following.

(a) The angle between planes *ABC* and *OAB* is $O\hat{D}C$

$$O\hat{D}C=Tan^{-1}\frac{C\sqrt{a^2+b^2}}{ab}$$

(b) $$ON=\frac{abc}{\sqrt{a^2b^2+b^2c^2+c^2a^2}}$$

12. Answer any two parts out of four parts (i), (ii),(iii) and (iv)

(i) (a) If $u = f(x, y)$ where $x = r^2 - s^2$ and $y = 2rs$, prove that

$$r\frac{\partial u}{\partial r} - s\frac{\partial u}{\partial s} = 2[r^2 + s^2]\frac{\partial u}{\partial x}$$

(b) The radius of a cylinder increases at the rate of 0.2 cm/sec while the height decreases at the rate of 0.5 cm/sec. Find the rate at which the volume is changing at the instant when r =8 cm and h = 12cm.

(ii) Consider the system of equation

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

Use the Gauss seidal iterative method and perform three iterations. Take initial vectors as $x_1^0 = x_2^0 = x_3^0 = 0$

(iii) Express sin (x+h) as a series of powers of h and evaluate sin 44^0 correct to 5 decimal places.

(iv) (a) Show that, f(x) = x + sinx – 1, has root between x = 0 and x = π/2.

(b) Use Newton Raphson method, with x_0 =0.5 as initial value to calculate a more accurate value of the root to the above equation, with convergence to 3 decimal places.