THE OPEN UNIVERSITY OF SRI LANAKA

DIPLOMA IN TECHNOLOGY – LEVEL 04

FINAL EXAMINATION – 2010/2011

MPZ 4230 – ENGINERING MATHEMATICS II

DURATION: THREE (03) HOURS

Date: 15th March 2011 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer only six (06) questions.
- State any assumptions you required.
- Do not spend more than 30 minutes for any problem.
- Show all your workings.
- All symbols are in standard notation.

01. a) If $u = f\,(y-z,\, z-x,\, x-y)$

i. Find $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ *and* $\frac{\partial u}{\partial z}$

ii. Prove $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ *(marks 35)*

b) Find and classify the stationary points of the function $f(x, y) = x^2 - 4xy + y^3 + 4y$ *(marks 25)*

c) Let $\underline{F}(x,y) = (x+2y)\underline{i} + (x+y)\underline{j}$

i. Is this vector field conservative?

ii. Evaluate $\int_{(0,0)}^{(2,1)} \underline{F}.dr$ along straight line from (0, 0) to (2, 1).

iii. Evaluate $\int_{(0,0)}^{(2,1)} \underline{F}.dr$ along straight line from (0, 0) to (0, 1) and then to (2, 1).

iv. Give the comments about part (ii) and part (iii) have different values. *(marks 40)*

02. a) i. State the Cauchy Riemann equation for the function f(z) to be analytic.

ii. Show that the function $u = x^3 - 3xy^2$ is a harmonic function.

iii. Finding the complex conjugate v of $u = x^3 - 3xy^2$, obtain the analytic function $f(z) = u + iv$.

iv. Express $f(z)$ in terms of z. *(marks 60)*

b) i. State Cauchy's integral formula.

ii. Evaluate the integral $\int_c \frac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)(z-2)}$, where c is the circle $|z|=3$. *(Marks 40)*

03 a) Suppose that the life of a particular brand of watch battery is 1000 days on average with a standard deviation of 50. Suppose further that the lifetimes of these batteries are normally distributed.

i. What probability of these batteries will last more than 920 days?

ii. How long must a battery last so that it is in the top 10% of all batteries? *(marks 40)*

b) A quality control engineer is in charge of testing whether or not 90% of the DVD players produced by his company conform to specifications. To do this, the engineer randomly selected a batch of 12 DVD players from each day's production. The day's production is acceptable provided no more than 1 DVD players fails to meet specifications. Otherwise, the entire day's production has to be tested.

i. What is the probability that the engineer incorrectly passes a day's production as acceptable if only 80% of the day's DVD players actually conform to specification?

ii. What is the probability that the engineer unnecessarily requires the entire day's production to be tested if in fact 90% of the DVD players conform to specifications? *(marks 30)*

c) A life insurance salesman sells on the average 3 life insurance policies per week. Use Poisson distribution

i. Calculate the probability that in a given week he will sell some policies

ii. Assume that there 5 working days per week, what is the probability that in a given day he will sell one policy? *(marks 30)*

04. The Numbers of insured commercial bank y (in thousands) in the United States for the year 1987 to 1996 are shown in the table.

Year	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996
Y	13.70	13.12	12.71	12.34	11.92	11.46	10.96	10.45	9.94	9.53

a) Make a scatter plot of the data, letting x represent the number of years since 1987. *(marks 10)*

b) Estimate the regression line. *(marks 35)*

c) Find the standard error. *(marks 20)*

d) Test whether the slop parameter is zero.($\alpha = 0.1$) *(marks 20)*

e) Find 95% confidence interval for the slope parameter. *(marks 15)*

05. a) The CEO of large electric utility claims that at least 80 percent of his 1,000,000 customers are very satisfied with the service they received. To test this claim, the local newspaper surveyed 100 customers, using simple random sample. Among the sampled customers, 73 percent say they are very satisfied. Based on these findings, can we reject the CEO's claim that at least 80% of the customers are very satisfied? Use a 0.05 level of significance. (*marks 40*)

b) Suppose you are interested in developing a counseling technique to reduce stress within marriages. You randomly select two samples of married individuals out of ten churches in the association. You provide Group 1 with group counseling and study materials. You provide Group 2 with individual counseling and study materials. At the conclusion of the treatment period, you measure the level of marital stress in the group members. Here are the score:

Group 1								Group 2							
25	17	29	29	26	24	27	33	21	26	28	31	14	27	29	23
23	14	21	26	20	27	26	32	18	25	32	23	16	21	17	20
20	32	17	23	20	30	26	12	26	23	7	18	29	32	24	19

Here are the compute means and standard deviations.

	Group 1	Group 2
Mean	24.13	22.88
Standard Deviation	5.64	6.14

Are these groups significantly different in marital stress?
(Assume a significance level of 0.1) (*marks 60*)

06 Let $\frac{dy}{dt} = t + y \quad ; \quad y(0) = 1 \quad , \quad t \in [0,1]$

a) Find $y(0.1)$ and $y(0.2)$ by using

i. Euler Method. (*marks 20*)

ii. Runge – Kutta second order method. (*marks 20*)

iii. Taylor series method. (*marks 40*)

b) Find the exact solution of $y(0.1)$ and $y(0.2)$. (*marks 20*)

07. a) State the classification for a partial differential equation in to elliptic, parabolic and hyperbolic form. *(marks 10)*

b) Find the values of $u(x, t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial^2 t}$

with boundary conditions $u(0, t) = 0 = u(8, t)$ and $u(x, 0) = 4x - \frac{1}{2}x^2$

at the points.

$x = i$: i = 0, 1, 2, 3... 7 and

$t = \frac{1}{8}j$: j = 0, 1, 2, 3, 4. *(marks 90)*

08. a) A series RLC circuit is connected to a D.C source of emf. V volts. Using differential Equation, discuss the current flowing in circuit as a function of time t when the circuit is

i. over - damped

ii critically - damped

iii. under – damped

Kirchhoff's voltage law for series circuit yield

$$V_L + V_R + V_C = V_0$$

Where, $V_L = L\frac{di}{dt}$, $V_R = iR$, $V_C = \frac{1}{c}\int idt$ *(marks 70)*

b) In RLC series circuit inductance (L) = 10 H, Resistance(R) = 1000 Ω, capacitance (C) = 10 μF and Voltage (V_0) = 3 V. Assume that the circuit is initially at rest, find

i. Current at time T.

ii Capacitor voltage at time T. *(marks 30)*

09. a) i. Define a subspace of a vector space.

ii. Consider the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$.

Show that W = {x + 2y, y, -x + 3y) is a subspace of $\mathbb{R}^3$.

(marks 20)

b) Let V be a vector space over a field F and let S be a finite subset of V. Explain What is the meant by the following.

i. The set S is linearly independent over F.

ii. The set S is linearly dependent over F. *(marks 20)*

c) i. Show that the set of vectors (u_1, u_2, u_3) of $\mathbb{R}^3$ is linearly independent over $\mathbb{R}$, where $u_1 = (1, 2, 2)$, $u_2 = (1, -2, 2)$ and $u_3 = (1, 0, 1)$ *(marks 40)*

ii. Construct and orthogonal basis for $\mathbb{R}^3$ out of above (u_1, u_2, u_3) by the Gram-Schmidt process.

d) Suppose T is a linear transformation defines by T: $\mathbb{R}^2 \to \mathbb{R}^2$ and $T\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}2\\0\end{pmatrix}$ & $T\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$. Find $T\begin{pmatrix}0\\1\end{pmatrix}$ *(marks 20)*

10. Let $A=\begin{pmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{pmatrix}$

a) Find the eigen values of A. *(marks 25)*

b) Find the unit eigen vectors of A. *(marks 30)*

c) Obtain a matrix P such that $P^{-1}AP$ is a diagonal matrix. *(marks 15)*

d) If $S = P^{-1}AP$, state the special properties of S^n and S^{-1} *(marks 10)*

e) Using the above results, reduce the quadratic form Q(x) to form Q(y), where

$$Q(x) = 8x_1^2 - 12x_1x_2 + 4x_1x_3 - 8x_2x_3 + 7x_2^2 + 3x_3^2$$

$$Q(y) = a_1y_1^2 + a_2y_2^2 + a_3y_3^2$$ *(marks 10)*

f). Obtain the relationship between $x^T = (x_1, x_2, x_3)^T$ and $y^T = (y_1, y_2, y_3)^T$ *(marks 10)*

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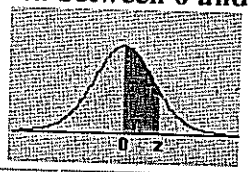
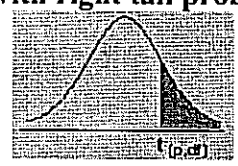
Standard Normal (Z) Table

Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905