THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY – LEVEL 04

FINAL EXAMINATION – 2011/2012

MPZ 4230 – ENGINEERING MATHEMATICS II

DURATION: THREE (03) HOURS

Date: 06th March 2012 **Time: 0930hrs – 1230hrs**

Instructions:

- Answer only six (06) questions.
- State any assumptions you required.
- Do not spend more than 30 minutes for any problem.
- Show all your workings.
- All symbols are in standard notation.

01. a) In surveying a triangular plot of land two of its sides were measured and found to be $a = 160\,\text{m}$ and $b = 210\,\text{m}$ in an error of at most $0.1\,\text{m}$ in either measurement ; the included angle was $C = \frac{\pi}{3}$ exactly. The length of the third side c is $\left(a^2 + b^2 - 2ab\cos C \right)^{1/2}$

i. Using appropriate Taylor polynomial estimate the value of the third side.

ii. Estimate the maximum error in calculating the length of the third side. *(marks 60)*

b) Let $\underline{F}(x,y) = x^2\underline{i} + y\underline{j}$

i. Is this vector field conservative?

ii. Evaluate $\int_{(2,0)}^{(0,2)} \underline{F}.d\underline{r}$ along the straight line from $(2,0)$ to $(0,2)$.

iii. Without integrating evaluate $\int_{(2,0)}^{(0,2)} \underline{F}.d\underline{r}$ along the quarter circle center at the origin and radius 2 between $(2,0)$ to $(0,2)$.

(marks 40)

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(marks 40)

04. Emission rate of carbon monoxide in grams per mile were taken to see if the emission rate increases with the mileage of a car. The carbon monoxide emission rate and the mileage were record on several cars of the same make and model.

Mileage $(\times 10^3)$	Emission
0	50
0	40
1	56
1.1	49
2	58
2.2	58
3	60
3	65
4	75
5.3	77
6	73
6	86

a) Make a scatter plot of these data on a graph paper. *(marks 10)*

b) Calculate the linear regression equation. Draw this on your plot. *(marks 25)*

c) Find the correlation coefficient between the mileage and the carbon monoxide emission rate. Comments your answer. *(marks 20)*

d) Find an estimate of the error variance. *(marks 20)*

e) Test whether the slope parameter is significantly zero. *(marks 15)*

f) Find 95% confidence interval for the slope of regression line. *(marks 10)*

05. a) i State and prove Euler's method for solving an ordinary differential equation.

ii Find $y(0.1)$ and $y(0.2)$, where $\frac{dy}{dx}=(1+x)y^2$ and $y(0)=1$.

iii Find the actual value of $y(0.1)$ and $y(0.2)$. *(marks 50)*

b) Using Taylor's series method (up to fourth order), solve $\frac{dy}{dx}=2y+3e^x$, $y(0)=0$ at $x=0.1$ and $x=0.2$.

(marks 50)

06. a) State the classification for a partial differential equation in to elliptic, parabolic and hyperbolic form. *(marks 10)*

b) Obtain the numerical solution to solve $\frac{\partial u}{\partial t} = 5\frac{\partial^2 u}{\partial x^2}$, under the conditions $u(0,t) = 0, u(8,t) = 100$ and

$$u(x,0) = \begin{cases} 20x & \text{for} \quad 0 < x \leq 5 \\ 100 & \text{for} \quad 5 < x \leq 8 \end{cases}$$

For five time step having $h = 1$ *(marks 90)*

07. a) A series RLC circuit is connected where there no externally applied voltage. Using differential equation, discuss the charge of the capacitor in circuit as a function of time t when the circuit is

i. over - damped

ii critically - damped

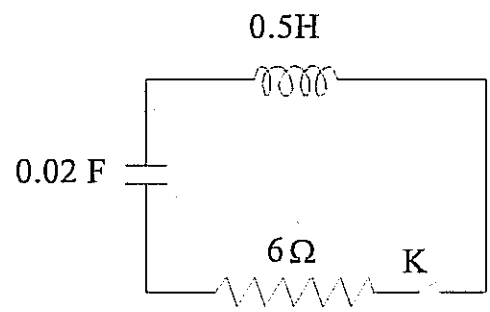
iii. under – damped

Kirchhoff's voltage law for series circuit yield

$V_L + V_R + V_C = V_0$

Where, $V_L = L\frac{di}{dt}$, $V_R = iR$, $V_C = \frac{1}{c}\int idt$, $i = \frac{dq}{dt}$ *(marks 60)*

b). An inductor of 0.5 H is connected in series with a resistor of 6Ω, a capacitor of 0.02 F and a switch K. Consider at time $t = 0$ the capacitor charge is q_0.



i. Set up the differential equation for the charge on the capacitor.

ii. Using above result find current and capacitor voltage at time T. *(marks 40)*

08. a) i. Show that the vectors $\underline{u}_1 = (1,1,1)$, $\underline{u}_2 = (1,2,3)$ and $\underline{u}_3 = (1,5,8)$ span $\mathbb{R}^3$.

ii. Also determine $\underline{u}_1 + \underline{u}_2 + \underline{u}_3$, $\underline{u}_1 + 3\underline{u}_2$ and $\underline{u}_2 + \underline{u}_3$ are linearly independent vectors.

iii. Apply the Gram – Schmidt orthogonalization process to find an orthogonal basis and the orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by $\underline{u}_1$, $\underline{u}_2$, $\underline{u}_3$. *(marks 60)*

b) i. Define a subspace of a vector space.

ii. Consider the vector space $\mathbb{R}^3$ over the field $\mathbb{R}$.

Show that $W = \{x + 2y, y, -x + 3y\}$ is a subspace of $\mathbb{R}^3$. *(marks 40)*

09. a) Suppose T is a linear transformation defined by $T: \mathbb{R}^2 \to \mathbb{R}^2$ and $T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}3\\2\end{pmatrix}$, $T\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$

Find a basis and the dimension of

i. The image of T

ii. The Kernel of T *(marks 30)*

b) Let $A = \begin{pmatrix}2 & -2\\ -2 & 5\end{pmatrix}$

i. Find the eigen values of A.

ii. Find the unit eigen vectors of A.

iii. Find an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix.

iv. If $S = P^{-1}AP$ state the special property of S and find S^{-1}..

v. Using the above results, reduce the quadratic form $Q(x)$ to form $Q(y)$, where

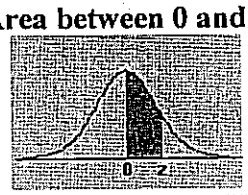
$$Q(x) = 2x_1^2 - 4x_1x_2 + 5x_2^2$$

$$Q(y) = a_1y_1^2 + a_2y_2^2$$

vi. Obtain the relationship between $x^T = (x_1, x_2)^T$ and $y^T = (y_1, y_2)^T$ *(marks 70)*

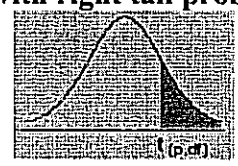
Standard Normal (Z) Table

Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

t table with right tail probabilities



df\p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
inf	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905