The Open University of Sri Lanka
B.Sc./B.Ed. Degree, Continuing Education Programme
Final Examination -2006/2007
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics - Paper II
Pure Mathematics/Computer Science – Level - 05

082

Duration :- Two and Half Hours

Date :- 02-12-2006. **Time :- 01.30 p.m. – 04.00 p.m.**

Answer Four Questions Only.

01. State whether the following statements are true or false. Justify your answers.

(i) $\exists y\ \forall x\ P(x, y) \Rightarrow \forall x\ \exists y\ P(x, y)$ where $P(x, y)$ is a statement about the variables x and y.

(ii) The negation of the statement "Nayani, Kamani and Ramani are tall girls" is equivalent to the statement "Nayani, Kamani and Ramani are short girls".

(iii) $\{x : x > 2 \text{ iff } x^2 > 4\} = (2, \infty)$.

(iv) There is a length r called the radius for each circle centered at the origin.

(v) The statement "(4 is prime) and ($\exists\ n \in \mathbb{N}$ such that $\sum_{k=1}^{n} \frac{1}{k} > 2006$)" is equivalent to the statement "$\exists\ n \in \mathbb{N}$ such that (4 is prime and $\sum_{k=1}^{n} \frac{1}{k} > 2006$)".

02. (i) Use the contra positive to show that if n is a natural number and $n^2 \geq 35$ then $n \geq 6$.

(ii) Prove that every group G with at least two elements has an abelian subgroup H with at least two elements.

(iii) Prove or disprove that, for each positive integer n, $n^2 + n + 17$ is prime.

03. (i) Consider the binary relation R defined on Z^+ by $x\ R\ y$ if $2x + 3y = 5$.

Is the relation R

(a) reflexive? (b) symmetric? (c) transitive? (d) an equivalence?

Explain your answer.

(ii) Does there exist a binary relation ρ defined on $S = \{1, 2, 3\}$ such that ρ is reflexive, symmetric and antisymmetric. Justify your answer.

(iii) For $A, B \in P(Z^+)$, the set of all subsets of Z^+, define the binary relation ρ by $A\ \rho\ B$ if $A \cup B = Z^+$. Show that ρ is symmetric. Is ρ transitive? Explain your answer.

04. (i) Let $\mathbb{Q}'$ be the set of all nonzero rational numbers. Let $*$ be defined in $\mathbb{Q}'$ as

$a * b = 7ab$ for each $a, b \in \mathbb{Q}'$. Show that $(\mathbb{Q}', *)$ is a group.

Prove or disprove that $\left(\left\{\frac{m}{7} : m \in Z - \{0\}\right\}, *\right)$ is a subgroup of $(\mathbb{Q}', *)$.

(ii) Let G be a group such that $(ab)^2 = a^2b^2$ for each $a, b \in G$.

Is G an abelian group? Justify your answer.

05. (i) In a certain batch of 203 university students, 64 take discrete mathematics, 112 take bio-mathematics, 86 take automata theory, 35 take both discrete mathematics and bio-mathematics, 32 take both bio-mathematics and automata theory, 28 take both automata theory and discrete mathematics and 187 take at least one of these three subjects.

What is the probability that:

(a) a student will take all of these three subjects?

(b) a student will take exactly two of these three subjects?

(c) a student will take exactly one of these three subjects?

(d) a student will not take any of these three subjects?

(ii) Prove by deduction that the pair of events $\{A, B\}$ is independent in a probability space (S, P) implies that the pairs of events $\{A^C, B\}$, $\{A, B^C\}$ and $\{A^C, B^C\}$ are also independent in the space (S, P).

06. Solve the following difference equations.

(i) $(n-1)f(n) - nf(n-1) = 7$.

(ii) $f(n+3) + f(n+2) - 4f(n+1) + 6f(n) = 0$.

(iii) $f(n+2) - f(n+1) - 12f(n) = n + 1$.