The Open University of Sri Lanka
B.Sc./B.Ed. Degree, Continuing Education Programme
Final Examination -2006/2007
PMU 3294/PME 5294/CSU 3276 – Discrete Mathematics - Paper I
Pure Mathematics/Computer Science – Level - 05

101

Duration :- Two and Half Hours

Date :- 02-12-2006. **Time :- 09.30 a.m. – 12.00 noon.**

Answer Four Questions Only.

01. (i) Let f be a function defined on a subset S of $\mathbb{R}$, the set of all real numbers, into $\mathbb{R}$. Consider the following statements about f:

(a) f is said to be increasing if for each $x, y \in S$, $x < y \Rightarrow f(x) < f(y)$.

(b) f is said to be decreasing if for each $x, y \in S$, $x < y \Rightarrow f(x) > f(y)$.

Write the meaning of the following.

(α) f is not increasing

(β) f is not decreasing.

(ii) Suppose $\{s_1, s_2, s_3\} \subseteq S$ with $s_1 < s_2 < s_3$. Is it true that, f is not increasing if and only if f is decreasing?

Justify your answer. Explain carefully whether your answer depends on the assumption that $|S| \geq 3$.

(iii) Let $S_0 = [0, 1]$ and $f_0(x) = \begin{cases} 3x+1 & \text{if } x \in [0,1) \\ 3 & \text{if } x = 1. \end{cases}$

Is f_0 increasing?

Is f_0 decreasing?

Prove your answer.

02. (i) $a_1, a_2, a_3, \ldots, a_n, \ldots$ is a sequence of numbers which satisfies the recurrence relation, $a_{n+2} - 7a_{n+1} + 12a_n = 0$ for each $n \geq 1$.

It is given that $a_1 = 7$. Prove by induction that $a_n = 3^n + 4^n$ for each $n \geq 1$.

(ii) Prove or disprove that if f is a function such that $|f|$ is continuous at 0 then f is continuous at 0.

(iii) Prove that, $\frac{1}{2}(a+b) \geq (ab)^{\frac{1}{2}}$ for each $a, b > 0$.

03. A bag contains six blue marbles, eight red marbles and ten white marbles.

(i) Find the number of ways that six marbles can be drawn from the bag.

(ii) Find the number of ways that six marbles can be drawn from the bag if two of the marbles must be blue, two of the marbles must be red and two of the marbles must be white.

(iii) Find the number of ways that six marbles of the same colour can be drawn from the bag.

(iv) Find the number of ways that six marbles can be drawn from the bag if two of the marbles must be of one colour and the remaining four marbles must be of a different colour.

04. Let $(G, *)$, $(G', *')$ be groups and let f be a homomorphism from G to G'.

(i) Prove that (a) $f(e) = e'$ where e, e' are identities of G, G' respectively.

(b) $f(g^{-1}) = [f(g)]^{-1}$ for every $g \in G$.

(ii) Let $\text{Ker} f = \{g \in G : f(g) = e'\}$. Show that $\text{Ker} f$ is a subgroup of G. Show also that $g^{-1}(\text{Ker} f)g = \text{Ker} f$ for each $g \in G$, where $g^{-1}(\text{Ker} f)g = \{g^{-1}xg : x \in \text{Ker} f\}$.

05. Let G be a graph with n vertices (where $n \geq 3$) and suppose that A is the adjacency matrix of G. Prove the property that G is connected if and only if every entry in the matrix $A + A^2 + A^3 + \cdots + A^{n-1}$ is nonzero.

The adjacency matrix A of a graph G is given by

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}.$$

Use the above property to determine the connectedness of G.

Is G a forest? Explain your answer.

06. Solve the following difference equations.

(i) $f(n + 2) - 4f(n + 1) + 4f(n) = n2^n$ given that $f(1) = 0$ and $f(2) = 1$.

(ii) $f(n) - nf(n - 1) = n!$.

(iii) $f(n + 2) + 2f(n + 1) + 4f(n) = 0$.