THE OPEN UNIVERSITY OF SRI LANKA
B.Sc/B.Ed Degree Programme,Continuing Education Programme.
APPLIED MATHEMATICS-LEVEL 05
AMU 3182/AME 5182-MATHEMATICAL METHODS I
FINAL EXAMINATION 2006/2007

DURATION: TWO AND HALF-HOURS

DATE:10-11-2006	Time:1.30pm-4.00pm

ANSWER **FOUR** QUESTIONS ONLY

(01) (a) let B be a matrix of order $n \times n$. Suppose that B has n linearly independent eigenvectors $\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n$ corresponding to the eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots\lambda_n$ respectively. Show that the general solution of the system of differential equations $\dot{X} = BX$ is $X = \sum_{r=1}^{n} c_r a_r e^{\lambda_r t}$ where c_r are arbitrary constants.

(b) Find the general solution of the system of simultaneous differential equations given below.

(i)
$$\dot{x}_1 = 7x_1 + 4x_2 - x_3$$
$$\dot{x}_2 = 4x_1 + 7x_2 - x_3$$
$$\dot{x}_3 = -4x_1 - 4x_2 + 4x_3$$

(ii) Given that x_1, x_2 and x_3 denote the position co-ordinates of a particle moving in three dimensions. If the particle moves along the plane $x_1 + x_2 + 2x_3 = 0$, show that two times the velocity in the x_3 direction is equal to the sum of the velocities in the x_1 and x_2 directions.

(02) (a) Solve each of the following boundary value problem.

(i) $u''(x) + 4u'(x) + 13u(x) = 26, \quad u\left(-\pi/2\right) = 0, u\left(\pi/2\right) = 0$

(ii)
$3x^2u''(x) - 2xu'(x) - 2u(x) = 0$ given that $u'(1) = 1$ and $\lim_{x \to \infty} u(x)$ is bounded

(iii)
$(1 - x^2)u''(x) - xu'(x) + 4u(x) = 0$ given that both $\lim_{x \to -1} u'(x)$ and $\lim_{x \to 1} u'(x)$ are bounded

(b) Find the general solutions of each of the following systems of simultaneous different equations.

(i)
$$\frac{dx_1}{dt} = x_1 + 2x_2 + e^t$$
$$\frac{dx_2}{dt} = -x_1 + 4x_2 + 2$$

(ii)
$$\ddot{x}_1 + 2\ddot{x}_2 + \dot{x}_1 + x_1 - 3x_2 = \sin t$$
$$3\ddot{x}_1 + \ddot{x}_2 + 2\dot{x}_2 + 2x_1 + x_2 = \cos t - 2\sin t$$

(03) (a) Solve the following differential equation.

$$9x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 0$$

(b) Use suitable change of variable and find the general solution of the differential equatio

$$2(x-\eta)^2\frac{d^2y}{dx^2} + 11(x-\eta)\frac{dy}{dx} - 5y = 0; \qquad x > \eta \quad \text{where } \eta \text{ is a constant}$$

(c) By eliminating the arbitrary constants P and Q, obtain differential equations satisfied each of the following functions.

(i) $y = \cos(x+P) + Qx$

(ii) $y = \dfrac{P}{x} + \dfrac{x}{Q}$

(04) (a) Using the integrating factor method, find the general solution of each of the following partial differential equations.

(i) $\dfrac{\partial u}{\partial y} - xyu = y,$ where u is a function of x and y

(ii) $\left(x^2+1+2x\right)\dfrac{\partial u}{\partial x} + 2u + 2xu = \left(x^2+2x+1\right)e^x,$ where $u = u(x,y)$

(iii) $\dfrac{\partial u}{\partial y} + \dfrac{1}{y^2(1+x)}u = -2(1-x)\exp\left(\dfrac{1}{y(1+x)}\right),$ $y \neq 0,$ $x \neq -1$ where $u = u(x,y$

(b) Find the general solution of the pair of simultaneous partial differential equations.

$$\frac{\partial u}{\partial y} = 2y + e^{y-x}$$

$$\frac{\partial u}{\partial x} = 2x - e^{y-x}$$

(c) Find the equation of the characteristic curves for the partial differential equation.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$$

Hence or otherwise, find the general solution of the given partial differential equation.

(05) (a) For each of the following partial differential equation, find the order of the equation and state whether it is linear or not. (In each case, $u=u(x,y)$)

(i) $\left(\frac{\partial u}{\partial x}\right)^2 + u = 0$

(ii) $\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = xy$

(iii) $\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^2 u}{\partial x \partial y} = 0$

(b) Find the general solution of each of the following equations.

(i) $\frac{\partial^2 u}{\partial y^2} - \frac{1}{y}\frac{\partial u}{\partial y} = 0 \qquad (y \neq 0) \quad \text{where } u = u(x,y)$

(ii) $\frac{\partial^2 u}{\partial x \partial t} + \frac{1}{t}\frac{\partial u}{\partial x} = x \qquad (t \neq 0) \quad \text{where } u = u(x,t)$

(iii) $\frac{\partial^2 u}{\partial x \partial y} = 0 \qquad \text{where } u = u(x,y)$

(c) Using the method of characteristics, find the general solution of the equation.

$$y^2\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - 4y^2\frac{\partial u}{\partial x} + \left(4y + \frac{1}{y}\right)\frac{\partial u}{\partial y} = 0 \quad (y \neq 0)$$

(06) (a) Write down the chain rule for partial derivatives.

(b) A function u(r,t) satisfies the equation

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right)=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$ where c is a constant.

By introducing the new dependent variable $v(r,t)=ru(r,t)$ and writing

$\xi=r+ct,\quad \eta=r-ct,$ reduce this equation to $\dfrac{\partial^2 v}{\partial\xi\partial\eta}=0$.

Hence, show that the general solution $u(r,t)$ has the form

$$u(r,t)=\frac{1}{r}\left[f(r+ct)+g(r-ct)\right]$$

Where f and g are arbitrary (twice differentiable) functions.

(c) Given that $u(p,q)=f(p)+g(p^2+2q^2)$ is the general solution of a partial differential equation, find the particular solution which satisfies the additional conditions.

$$u(p,p)=2p^2+q^4$$

$$\frac{\partial u}{\partial p}(p,p)=2p^3$$