The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Final Examination – 2006/2007
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

Duration :- Two and a Half Hours

Date :- 23-11-2006. **Time :- 1.00 p.m. – 3.30 p.m.**

Answer Four Questions Only.

01. Obtain, in a usual notation, the equation $\frac{d^2u}{d\theta^2}+u=\frac{P}{h^2u^2}$ for the motion of a particle moving under a central force P towards the centre O.

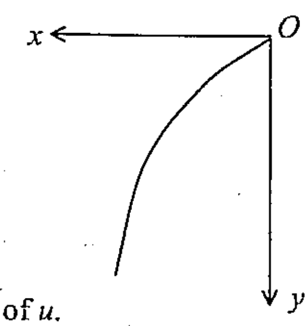
If $P=\mu/r^2$, where μ is a positive constant, and if the speed of the particle is V when it is at a distance r from O, show that the orbit is an ellipse, a parabola or a hyperbola according as $V^2-\frac{2\mu}{r}\lesseqgtr 0$.

A particle of mass m describing a circle of radius c under an attractive force μ/r^2 per unit mass towards the centre, collides and coalesces with a particle of mass λm which is at rest. Show that the orbit of the combined particle is an ellipse with major axis $c\ \mathrm{cosec}^2\alpha$, where $\sec^2\alpha=2(1+\lambda)^2$.

02. Obtain the velocity and acceleration components in intrinsic coordinates, for a particle moving along a curve in a plane.

The cross-section of a smooth surface has the equation:

$$y=a\left[\cosh\left(\frac{x}{a}\right)-1\right]$$



where the x-axis is horizontal and the y-axis is vertically downwards as shown in the diagram.

A particle of mass m is projected from O with a horizontal speed of u.

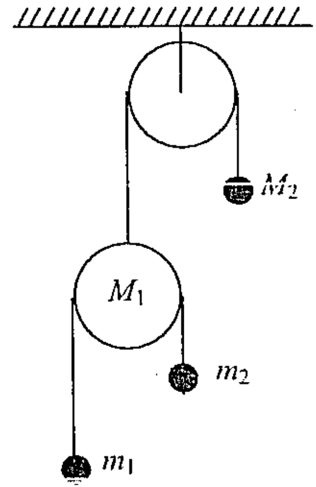
(a) Show that if $u^2>ga$ the particle will leave the surface immediately.

(b) Show that if $u^2<ga$ the particle will leave the surface when $y=a-\frac{u^2}{g}$.

03. In the usual notation, derive Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j,\quad j=1,2,\ldots,n;$$ where T is the kinetic energy of the system.

A mass M_2 hangs at one end of a string which passes over a fixed frictionless non-rotating pulley. At the other end of this string there is a non – rotating pulley of mass M_1 over which there is a string carrying masses m_1 and m_2 as shown in the following figure.



(a) Set up the Lagrangian of the system.

(b) Find the acceleration of mass M_2.

04. Obtain, in the usual notation, the equation $\frac{\partial^2 r}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

An object is projected vertically downward with speed v_0. Prove that after time t, the object is deflected east of the vertical by an amount $\omega v_0 \cos\lambda\, t^2 + \frac{1}{3}\omega g t^3 \cos\lambda$, where λ is the latitude of the point of projection and ω is the angular speed of earth about its polar axis.

If the point of projection is at a height h above the earth, show that the particle will reach at a point east of the vertical at a distance $\frac{\omega\cos\lambda}{3g^2}\left(\sqrt{v_0^2+2gh}-v_0\right)^2\left(\sqrt{v_0^2+2gh}+2v_0\right)$.

05. Derive Euler's equations of motion of a rigid body rotating about a fixed point.

The principal moments of inertia of a body at the centre of mass are A, $3A$, $6A$. The body is initially rotated with an angular velocity having components about the principal axes $3n$, $2n$, n respectively. In the subsequent motion under no force, if ω_1, ω_2, ω_3 denote the angular velocities about the principal axes at that time t, show that

$\omega_1 = 3\omega_3 = \frac{9n}{\sqrt{5}}\operatorname{sech} u$ and $\omega_2 = 3n \tanh u$, where $u = 3nt + \frac{1}{2}\ln 5$.

06.(i)(a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(b) Using Hamilton's equations find the equations of motion of a projectile in three dimensional space.

(ii)(a) Define canonical transformation.

(b) Show that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical.