The Open University of Sri Lanka
B.Sc./B.Ed. Continuing Education Degree Programm
Final Examination -2006/2007
AMU3183/ AME5183- Numerical Methods II

0142

Duration : Two and Half Hours

Date: 02/11/2006 Time: 1.00p.m.-3.30p.m.

Answer Four Questions And no More

(1) (a) Explain how you would find the Larangian interpolation polynomial $p(x)$ for the data set $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$.

(b) With the usual notation, prove that the error of interpolation by Lagrange's method is $\dfrac{\pi(x) f^{n+1}(c)}{(n+1)!}$, where $c \in (x_0, x_n)$.

(c) A census of the population of the united states is taken every 10 years.The following table lists the population, in thousands of people,from 1940 to 1990.

year	1940	1950	1960	1970	1980	1990
population (in thousands)	132,165	151,326	179,323	203,542	226,542	249,1623

Estimate the population in 2000 and 1930 using Lagrangian interpolation formula.

(2) In the usual notation, centered difference operator δ is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$.Prove the following.

(a)

(i) $\Delta = \dfrac{\delta^2}{2} + \delta\sqrt{1 + \dfrac{\delta^2}{4}}$, where Δ is the forward difference operator.

(ii) $\nabla = -\dfrac{\delta^2}{2} + \delta\sqrt{1 + \dfrac{\delta^2}{4}}$, where ∇ is the backward difference operator.

(b) If average operator μ is defined as $\mu = \dfrac{1}{2}\left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)$,

show that $\mu\delta = \dfrac{1}{2}(\Delta + \nabla) = \dfrac{1}{2}\Delta E^{-1} + \dfrac{1}{2}\Delta$

(c)Complete the following difference table

k	y_k	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	....				
1	....	1.38×10^{-4}			
2	....	....	....		
3	....	....	....	1×10^{-6}	
4	0.497621	....	-1×10^{-6}	....	-3×10^{-6}

Using Newton's forward difference formula, find the interpolating polynomial which fits best for the given data.

(3) (a) Derive normal equations for the least squares straight line fit.

(b)The height H and the quantity Q of water flowing per second are related by the law $Q=CH^n$,where C and n are constants. The quantity of water Q for seven different heights H are presented in the accompanying table.

H(*ft*)	1.2	1.4	1.6	1.8	2.0	2.4	2.6
Q(ft^3)	4.2	6.1	8.5	11.5	14.9	23.5	27.1

(i) Find the best values of C and n .

(ii) Estimate the value of Q corresponding to H=3 (*ft*).

(4) (a) Write down with usual notation ,Simpson's rule to evaluate $\int_a^b f(x)dx$.

(b) Prove that the truncation error E in using Simpson's rule for the integral $\int_a^b f(x)dx$ is given by $E=-\dfrac{(b-a)h^4 f^{iv}(c)}{180}$ where $c\in(a,b)$.

(c)How many subintervals should be taken in the interval (0,10) in order that the integral $\int_0^{10}\dfrac{dx}{1+x^2}$ is to be calculated accurate to two decimal places with the use of Simpson's rule?

(5) (a) Find coefficients a,b,n,m in Order that Rungu-Kutta formulae

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + mh, y_0 + nk_1)$$

$$y_1 = y_0 + ak_1 + bk_2$$

in solving $\frac{dy}{dx} = f(x, y)$.

(b) Determine y when x=0.05 and x =0.10 by the second order Rungu-Kutta method given that

$$\frac{dy}{dx} = x^2 + y^2, y(0) = 1$$

Round off your results to the appropriate decimal place.

(6) (a) Derive the Taylor's formula for finding solutions of a first order differential equation with a given initial value.

(b)Derive the Euler's method from the Taylor's formula.

(c) Determine y at x=0.05 and at x =0.10 by Euler method given that the function y=$y(x)$ satisfies $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.