THE OPEN UNIVERSITY OF SRI LANKA
B.Sc/B.Ed Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS - LEVEL 04
AMU2182/AME 4182 - STATISTICS I
FINAL EXAMINATION 2006/2007

DURATION: TWO AND HALF-HOURS

DATE: 13 – 11 – 2006	TIME: 09.30am – 12.00noon

ANSWER **FOUR** (4) QUESTIONS ONLY.

Statistical Tables are provided. Non-programmable calculators are permitted.

1. (a) A coin is biased so that the probability of getting a head is $\theta(0<\theta<1)$. This coin is tossed four times and the probability of getting at least one head is 0.8704.

   (i) Compute the value of θ.
   (ii) Find the number of times the coin should be tossed so that probability of getting no heads is less than 0.03.

   (b) A bag contains 3 coins; two fair coins and the biased coin described in part (a) above. One coin is chosen at random from this bag and tossed two times. If head turns up each time find the probability that the selected coin is the biased one.

2. A game is played as follows. A fair coin is tossed once. If it comes up heads, a regular die is rolled and the player will get a score equal to the face value of the die. If the coin comes up tails, two regular dice are rolled and the player will get a score equal to the sum of the face values of the two dice. Let X denote the score of the game.

   (a) Write down the probability distribution of X.

   (b) Find the probability of obtaining
   (i) an even score
   (ii) a score of at least 7
   (iii) a score greater than 5 but less than 10.

   (c) What is the expected score of the game?

3. The random variable X denotes the number of goals scored by a football team in a football match. Suppose that X follows a Poisson distribution with parameter λ so that the probability density function of X is given by

$$P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!} \quad x=0,1,2,\cdots.$$

(a) Show that the moment generating function $M_X(t)$ of X is given by $M_X(t)=e^{\lambda(e^t-1)}$.

(b) Using part (a) or otherwise find the expected value of X.

(c) In the past, this team was found to score at least one goal in 26 matches out of 40 matches they played. Estimate the average number of goals scored for a match.

(d) The football team will receive Rs.100000/- for playing a match. Further they will receive Rs.10000/- if they score 1 goal, Rs.15000/- if they score two goals and Rs.20000/- if they score 3 or more goals. Find the expected gain of the team.

4. In order to grow a certain tree, the pH value of the soil should be less than 6.6. A researcher, who is interested in growing these trees in a certain area, planted 1000 trees. Suppose the pH value of the soil sample taken from this area is normally distributed with mean 6.0 and standard deviation 2.0

a) Estimate the proportion of trees that will grow successfully in this area.

b) If the researcher wants 80% of trees to grow successfully what should be the average pH value of the soil in this area assuming the standard deviation is 2.0?

c) In order to test the pH value, researcher selected 16 soil samples randomly from this area. Find the probability that

(i) exactly 10 soil samples will have a pH value grater than 6.6.
(ii) their average pH value will exceeds 6.3.

5. Let random variable X denotes the lifetime (in years) of a certain electronic machine. Suppose the distribution function $F(x)$, of X is given by

$$F(x)=\frac{x(kx+8)}{(x+4)^2} \quad x>0,\ k>0$$

a) Compute the value of k.

b) Find the probability density function of X.

c) Find the probability that
   (i) machine will work at most 3 years.
   (ii) machine will work more than 5 years given that it has already worked more than 2 years.

d) What is the expected lifetime of the machine?

6. Suppose that two dimensional continuous random variables X and Y have a joint probability density function $f(x,y)$, given by

$$f(x,y)=kx\ (x+y), \qquad 0\le x\le 1,\ 0\le y\le 2$$

a) Find the value of k.

b) Find $E(XY)$, the expectation of XY.

c) Find the conditional density function of X given Y. Are X and Y independent? Give reasons for your answer.

d) In the usual notation, find

   (i) $P(0<X<0.5,\ 1<Y<2)$
   (ii) $P(X>Y)$