The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 04
Final Examination – 2006/2007
Applied Mathematics
AMU 2181/AME 4181 – Mathematical Modelling I

093

Duration :- Two and Half Hours.

Date :- 24-11-2006. **Time :- 9.30 a.m. – 12.00 noon.**

Answer Four Questions Only.

01. The ABC electric appliance company produces two products: refrigerators and ranges. Production takes place in two separate departments. Refrigerators are produced in department I and ranges are produced in department II. The company's two products are produced and sold on a weekly basis. The weekly production cannot exceed 25 refrigerators in department I and 35 ranges in department II, because of limited available facilities in the two departments. The company regularly employs a total of 60 workers in the two departments. A refrigerator requires 2 man-weeks of labour, while a range requires 1 man-week of labour. A refrigerator contributes a profit of Rs. 60 and a range contributes a profit of Rs. 40. Formulate the problem as a linear programming problem to determine the number of units of refrigerators and ranges should the company produce to realize a maximum profit. Solve the model and find the solution.

02. Find the solutions to the following problems using graphical method:

(a) Minimize $Z = -x_1 + 2x_2$,

subject to
$$-x_1 + 3x_2 \leq 10$$
$$x_1 + x_2 \leq 6$$
$$x_1 - x_2 \leq 2$$
$$x_1, x_2 \geq 0.$$

(b) Maximize $Z = -x_1 + 4x_2$

subject to
$$3x_1 - x_2 \geq -3$$
$$-0.3x_1 + 1.2x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

03. Solve the following linear programming problem using Big-M method:

Maximize $Z = 2x_1 - 3x_2 - 5x_3$

subject to
$$-2x_1 + 3x_2 \geq 5$$
$$2x_1 + 4x_2 - x_3 \leq 2$$
$$x_1 + 2x_3 \geq 3$$
$$x_1, x_2, x_3 \geq 0.$$

04.(a) What is meant by dual problem of a linear programming model?

(b) Consider the problem

Minimize $Z = 2y_1 + y_2$

subject to $y_1 - y_2 - y_3 \geq 5$

$y_1 + 2y_2 + 4y_3 \geq 8$

$y_1, y_2, y_3 \geq 0.$

What is the dual of the above problem? Find solution of the primal problem by solving its dual.

05. Consider the problem

Maximize $Z = 45x_1 + 100x_2 + 30x_3 + 50x_4$

subject to

$7x_1 + 10x_2 + 4x_3 + 9x_4 \leq 1200,$

$3x_1 + 40x_2 + x_3 + x_4 \leq 800,$

$x_1, x_2, x_3, x_4 \geq 0.$

The optimal tableau for this problem is given below:

Basis	x_1	x_2	x_3	x_4	x_5	x_6	Value
x_3	$5/3$	0	1	$7/3$	$4/15$	$-1/15$	$800/3$
x_2	$1/30$	1	0	$-1/30$	$-1/150$	$2/75$	$40/3$
Z	$-25/3$	0	0	$-50/3$	$-22/3$	$-2/3$	

If a new variable x_7 is added to this problem with a column $\begin{bmatrix} 10 \\ 10 \end{bmatrix}$ and corresponding cost coefficient $(C_7) = 120$, find the change in the optimal solution.

06. A company wants to produce three products A, B and C. The unit profits on these products are Rs. 4, Rs. 6 and Rs. 2 respectively. These products require two types of resources, man-power and material. The following linear programming problem is formulated for determining the optimal product mix:

Maximize $Z = 4x_1 + 6x_2 + 2x_3$

subject to $x_1 + x_2 + x_3 \leq 3$ (man-power)

$x_1 + 4x_2 + 7x_3 \leq 9$ (material)

$x_1, x_2, x_3 \geq 0,$

where x_1, x_2, x_3 are the number of units of the products A, B and C produced respectively.

The optimal simplex table for the above problem is given below:

basis	x_1	x_2	x_3	x_4	x_5	values
x_1	1	0	–1	4/3	–1/3	1
x_2	0	1	2	–1/3	1/3	2
z	0	0	–6	–10/3	–2/3	–16

(i) Find the range of the cost coefficient (C_3) of the non-basic variable x_3 such that the current optimal product mix remains optimal.

(ii) What happens if C_3 is increased to Rs. 12? What is the new optimal product mix in this case?

(iii) Find the range of the cost coefficient of the basic variable x_1 such that the current optimal product mix remains optimal.