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THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SOFTWARE ENGINEERING
FINAL EXAMINATION – 2009/2010
MPZ 4140/MPZ 4160 - DISCRETE MATHEMATICS I

DURATION – THREE (03) HOURS

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DATE : 08th May 2010	**TIME: 9.30 –12.30 hours**

Instructions:

- Answer only six questions.
- State any assumption that you required.
- Show all your workings.
- Please answer a total of six equations choosing at least one from each single section.

SECTION - A

01. i. Define a statement and decide which of following are proposition.

    a. 128 is an odd integer
    b. all triangles have three sides
    c. where are you going?
    d. Put the homework on the blackboard.

    ii. Let P be "He is tall" and let q be "he is handsome". Write each of the following statements in symbolic form using p and q.

    a. He is tall but not handsome.
    b. It is false that he is short or handsome.
    c. He is tall, or he is short and handsome.
    d. It is not true that he is short or not handsome.

iii. Construct the truth tables of the following compound statements.

a. $(p \vee q) \Rightarrow [\{p \wedge (\neg q)\} \Rightarrow (q \wedge p)]$

b. $[(p \wedge q) \Rightarrow r] \Rightarrow [p \Rightarrow (q \Rightarrow r)]$

iv. Use the laws at the algebra of propositions to show that

a. $[(p \vee q) \wedge (\sim p)] \Leftrightarrow [(\sim p \wedge q)]$

b. $(p \wedge q) \vee (\sim (\sim p) \vee q)) \Leftrightarrow p$

02. i. Using mathematical induction prove that

a. $6^n - 5n + 4$ is divisible by 5 for all $n \geq 1$

b. $n^2 > 2n+1 \quad ,\forall n \geq 3$

ii. Find a counter example to the following statement.

f $n = p^2 + q^2$ where p and q are prime then n is prime.

iii. If x^2 is an odd integer, then x is an odd integer. (use the contrapositive of this statement)

03. i. Prove De Morgan's Laws by using truth tables.

ii. Give the converse and contrapositive of each of the following implications.

a. If all mice eat cheese, then either the moon is blue or cows fly.

b. That Mary is happy implies that she has passed mathematics.

iii. Test the validity of the following argument.

a. "If Perera is a Doctor then he is clever. He is clever and rich. Therefore if he is rich then he is a Doctor.

b. "If I study, the I will not fail mathematics. If I do not play basketball, then I will study. But I failed mathematics. Therefore I played basketball"

SECTION - B

04. i. Let U be the set of letter of the alphabet. Let A = {a, b, c, ………, l}, B={h,i,d,……….,q}, and c = {o,p,q,……z}. Find the elements in each of the following sets.

a. $A \cup C$ b. $A' \cap B'$

ii. Without using venn diagram show that $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$

iii. Using the principle of duality show that

a. $A \cup (A \cap B) = A$

b. $(A \cap B) \cup (A \cap B^C) = A$

05. i. Define a function from a set A into a set B

ii. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} x^2 + 1 & if \ x > 0 \\ 2x + 1 & if \ x \leq 0 \end{cases}$$

And $g : R \to R$ be defined by

$$g(x) = \begin{cases} x^3 & if \ x > 0 \\ 3x - 7 & if \ x \leq 0 \end{cases}$$

Find $f \circ g$ and $g \circ f$

iii. Let $A = \left\{ x \mid x \neq \frac{1}{2} \right\}$ and $f : A \to R$ by $f(x) = \frac{4x}{2x-1}$. If f one-to-one? Find $f^{-1}(x)$, $dom \ f^{-1}(x)$ and range of $f^{-1}(x)$

iv. Prove that if f is a bijection, then $(f^{-1})^{-1} = f$.

v. On what condition can a constant function be,
   a. one-to-one function
   b. an onto function.

06. i. Explain the main difference between an ordered pair (a,b) and the set {a,b} with two elements.

ii. Find example of set and relation on the set which is symmetric but not reflexive or transitive.

iii. Show that the relation of congruence modulo m, $a \equiv b(\bmod m)$ in the set Z is an equivalence relation. That is the relation $R = \{(a,b)|\ (a-b) = km$ for some fixed integer m and $a,b,k \in Z\}$ is an equivalence relation.

Hint. Prove that $a \equiv b(\bmod m)$ is an equivalence relation on the integers, for any given value of n)

iv. For integers a, b define a ~ b if and only if 2a + 3b = 5n for some integer n. Show that ~ defines an equivalence relation on Z.

SECTION - C

07. i. Define what mean by $a|b$, where a,b are integers.

ii. Show that if $x|y$ then $x|\ |y|$

iii. Prove that $6|n^3 - n$, for all integers n.

iv. If $3|(2x+1)$ prove that $3|(4x^4 - 2x^3 - 10x^2 - 4x)$

v. Define prime number and prove that an integer $n \geq 2$ is composite if and only if it has factor x and y such that $1 < x < n$ and $1 < y < n$.

08. i. Define the greatest common divisor and least common multiple for a given non zero integers and b.

ii. If $\gcd(a,m) = \gcd(b,m) = 1$, then show that $\gcd(ab,m) = 1$.

iii. Find gcd(4034,13402) and find integers x, y that satisfy the linear equation 4034 x + 13402 y = 32

09. i. The linear equation $ax + by = n$ has a solution if and only if $g|c$, where $g = \gcd(a,b)$. If x_0, y_0 is any particular solution of this equation, then show that other solutions are

$$x = x_0 + \left(\frac{b}{g}\right)t, \quad y = y_0 - \left(\frac{a}{g}\right)t,$$

Where t is an arbitrary integer.

ii. Determine all solutions in the positive integers of the following diplomatine equation.

54x + 21y = 906

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