00264

THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SOFTWARE ENGINEERING /
DIPLOMA IN TECHNOLOGY – LEVEL 04
FINAL EXAMINATION – 2010/2011
MPZ 4140/MPZ 4160 - DISCRETE MATHEMATICS I

DURATION – THREE (03) HOURS

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DATE : 07th March 2011 **TIME: 9.30 –12.30 hours**

Instructions:

- Answer only six questions.
- State any assumption that you required.
- Show all your workings.
- Please answer a total of six equations choosing at least one from each single section.

SECTION – A

01. i. Which of the following statement are proposition? What are their truth values?

   a) $3+3=6$
   b) It will rain tomorrow
   c) Solve the following equation for x.
   d) The number 7
   e) 3 is an odd number and $Sin\frac{\pi}{6}=\frac{1}{\sqrt{2}}$

ii. Construct the truth table for each of the following statements.

   a) $[(p \Rightarrow q) \wedge (p \Rightarrow r)] \Rightarrow (q \Rightarrow r)$

   b) $[p \Rightarrow (q \Rightarrow r)] \Rightarrow [(p \wedge q) \Rightarrow r]$

iii. Let p,q,r be 3 statements. Show that $[(p \Rightarrow q) \wedge (p \Rightarrow r)] \Leftrightarrow [p \Rightarrow (q \wedge r)]$ is a tautology.

iv. Use the laws of the algebra of proposition to show that

a) $\neg(p \vee q) \vee (\neg p \wedge q) \equiv \neg p$

b) $\neg(p \wedge q) \wedge p \equiv (\neg q \wedge p)$

02. i. Give the converse and contrapositive of the statement "If you earn an A in logic, then I will buy you a new car".

ii. Test the validity of the following arguments.

a) "If 7 is less than 4, then 7 is not a prime number. 7 is not less than 4. Therefore 7 is a prime number".

b) "If it rains, then Pasindu will be sick. Pasindu was not sick. Therefore it did not rain.

iii. Give counter examples to prove the following existential statements.

a) $\exists x \in \mathbb{R},\ x^3 + x^2 - 2 = 0$

b) $\exists r \in Q,\ Sin(\pi r) = \frac{1}{2}$, where Q is the set of all rational numbers.

iv. Prove that the following statement is false.

$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R},\ \ x - y^2 = 15$

03. i. Using the mathematical induction prove that $f(n) = n(n^2 + 5)$ is divisible by 6 for all positive integer n.

ii. Write the statement: "Every student is hardworking or lucky passes exam" in predicate logic.

iii. Prove that, if n^2 is an even integer then n is an even integer. (Use the contra positive of this statement)

iv. Prove or disprove the following statement.

"The sum of any conseative integer is drivable by 5"

SECTION – B

04. i. Write following sets in a set-builder form.

a) $A=\{1, 3, 5, 7, ...\}$

b) $B=\{c, o, r, e, t\}$

ii. Let $A=\{r,s,t,u,v,w\}$, $B=\{u,v,w,x,y,z\}$, $C=\{s,u,y,z\}$, $d=\{u,v\}$, $e=\{s, u\}$ *and* $F=\{s\}$. Let X be an unknown set. Determine which sets A,B,C,D,E or F can equal X if we are given the following information.

a) $X \subset A$ *and* $X \subset B$

b) $X \not\subset A$ *and* $X \not\subset C$

iii. Find power set of set $G=\{5,\{2, 7\}\}$

iv. Without using venn diagram, show that $(A \cup B)' = A' \cap B'$

v. Let $A=\{a,b,c,d,e,f\}$, $B=\{d,e,f,g,h,i\}$, $C=\{b,c,e,g,h\}$
Find

$$(A \oplus B) \oplus (B \oplus C)$$

05. i. a) Define a composition of function.

b) Let

$$f(x)=\begin{cases} 2x^2 & ;x \geq 0 \\ x+6 & ;x<0 \end{cases} \quad \text{and}$$

$$g(x)=x+3, \quad x \in R.$$

Find $fog(x)$ *and* $gof(x)$

ii. Let $A=\{a,b\}$, $B=\{1, 2, 3\}$ find $A \times B$ *and* A^3

iii. Prove $(A \times B) \cap (A \times C) = A \times (B \cap C)$

06. i. Let R be the relation on $A=\{1,2,3,4\}$ defined by
$R=\{(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)\}$

Show that

a) R is neither reflexive, nor transitive
b) R is neither symmetric, nor antisymmetric.

ii. Give the example of relation R_1 on $B=\{1,2,3\}$ having the stated property: R_1 is both symmetric and antisymmetric.

iii. Let A be a set of integers and let ~ be the relation of A x A defined by $(a,b) \sim (c,d)$ if $ad = bc$. Prove that ~ is an equivalence relation.

iv. Define a partial order on a set X. X is a non empty set and $Y = p(x)$ is the power set. (i.e the set of all subject of X). For $A, B \in Y$, the relation $\boldsymbol{R}$ is defined by
$A\,\boldsymbol{R}\,B \Leftrightarrow A \subseteq B$
Prove that $\boldsymbol{R}$ is a partial order on Y.

SECTION – C

07. i. Show that if $a|b$ and $b|a$ the $a = \pm b$

ii. Let gcd (a,b)=1, prove that
$gcd\,(a+b,\, a^2 - ab + b^2) = 1 \text{ or } 3$

iii. Define a prime number.
a) If $n \geq 5$ is a prime number, show that $n^2 + 2$ is not a prime.

b) If $b(b \neq 2)$ is a prime number, show that
$b^2 + (b+2)^2 + (b+4)^2 + 1$ is divisible by 12.

08. i. Define greatest common divisor (gcd) and least common multiplier (lcm)

ii. Find gcd (198, 372, 510)

iii. Determine integers x and y such that

gcd (427, 3078) = 427x + 3078 y

Hence give the general solution of the above equation in integers x and y.

09. i. Prove that

a) If $\equiv b(\bmod m)$ then $a+c \equiv b+c(\bmod m)$ *and* $ac \equiv bc(\bmod m)$

b) $ak \equiv bk(\bmod m)$ *then* $a \equiv b\left[\bmod \dfrac{m}{(k,m)}\right]$

c) $a \equiv b(\bmod m)$, $b \equiv a(\bmod m)$ *and* $(a-b) \equiv o(\bmod m)$ are equivalent statements.

ii. Solve the following set of congruence simultaneously (Chinese Remainder Theorem)

$x \equiv 1(\bmod\ 3)$
$x \equiv 2(\bmod\ 5)$
$x \equiv 1(\bmod\ 7)$
$x \equiv 3(\bmod\ 11)$