

THE OPEN UNIVERSITY OF SRI LANKA
 DIPLOMA IN TECHNOLOGY / BACHELOR OF
 SOFTWARE ENGINEERING – LEVEL 04
 FINAL EXAMINATION – 2011/2012
 MPZ4140 / MPZ4160 - DISCRETE MATHEMATICS I



DURATION – ONE AND QUARTER (01-1/4) HOURS

Index. No.....

DATE : 27th February 2012

TIME: 130 – 1630 hrs.

Instructions:

- Answer only six questions.
- Please answer a total of six questions choosing at least one from each single section.
- State any assumption that you made.
- All symbols are in standard notation.

SECTION – A

01. i. Define a statement and which of the following are statements? What are their truth values?
- a) “Mahaweli is the longest river in Sri Lanka”
 b) “ $1 = 0$ ”
 c) “The number 7”
 d) “If I am Buddha, then I am not Buddha.”
- (20 marks)
- ii. Let P be “Pasindu is rich” and let q be “Pasindu is happy” Write each of the following in symbolic form.
- a) “Pasindu is poor but happy”
 b) “Pasindu is neither rich nor happy”
 c) “Pasindu is poor or else he is both rich and unhappy”
- (20 marks)
- iii. Determine the truth value of each of the following statements.
- a) “If $7 < 3$, then $-3 < -7$ ”
 b) “ $2 + 2 = 4$ if and only if $4 + 4 = 12$ ”
 c) “If $2 + 2 = 4$, then $3 + 3 = 8$ and $1 + 1 = 2$ ”
 d) “If $3 + 3 = 6$, then $4 + 4 = 9$ if and only if (iff) $1 + 1 = 4$ ”
- (20 marks)

- iv. Prove De Morgan's Laws by using truth tables. (20 marks)
- v. Let p, q, r be 3 statements. Show that $[p \rightarrow q] \Rightarrow [(p \wedge r) \rightarrow (q \wedge r)]$ is a tautology. (20 marks)
02. i. Give the converse and contrapositive of the following statements.
 a) "If I am Buddha, then I think"
 b) "If 2 is an odd number, then all primes are odd" (20 marks)
- ii. Test the validity of the following argument.
 "If two sides of a triangle are equal, then the opposite angles are equal. Two sides of a triangle are not equal. Therefore the opposite angles are not equal". (30 marks)
- iii. Determine the truth value and Negation of each of the following statements (all $x \in \mathbb{R}$ (real number)).
 a) $\forall x, |x| = x$
 b) $\forall x, x+1 > x$ (20 marks)
- iv. Find a counter example for each of the following statements.
 a) $\exists n \in \mathbb{N}, 6n+1$ and $6n-1$ are not primes.
 b) $\exists k \in \mathbb{R}, \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right) = k$ (10 marks)
- v. Prove that there exists a unique real number x such that $x^3 - 8 = 0$. (Use the direct proof). (20 marks)
03. i. Show that $p \wedge (p \vee q) \equiv p$, by using laws of the algebra of propositions. (20 marks)
- ii. Using mathematical induction prove that, for each positive integers.
 a) $n^3 + 2n$ is divisible by 3.
 b) $\sum_{r=1}^n r(r+1) = \frac{1}{3}n(n+1)(n+2)$. (60 marks)

iii. Transcribe the following into logical notation. Let the universe of discourse be the real numbers.

a) For any value of x , x^2 is nonnegative.

b) For every value of x , there is some value of y such that $x \cdot y = 1$.

(20 marks)

SECTION – B

04. i. Let $A = \{x : x \text{ is an odd integer}\}$,
 $B = \{x : x \text{ is an integer divisible by } 10\}$
 $C = \{x : x \text{ is an integer divisible by } 5\}$ and
 $D = \{x : x \text{ is an integer divisible by } 2 \text{ or } 5\}$

Find the elements in each of the following sets.

a) A, B, C, D

b) $A \cap D$

c) $B \cap C$

(30 marks)

ii. Without using venn diagram show that

$$A \cup (B \oplus C) = (A \cup B) \oplus (A \cup C)$$

$$\{\text{Where as } x \oplus y = (x \setminus y) \cup (y \setminus x)\}$$

(50 marks)

iii. Let $X = \{1, 2, 3, 4, 5\}$, $Y = \{3, 4, 5, 6, 7, 8\}$
and $Z = \{2, 4, 6, 7, 8, 9\}$. Find

a) $X \cup (Y \oplus Z)$

b) $X \oplus (Y \cap Z)$

(20 marks)

05. i. Let R be the set of all real numbers and let, $f : R \rightarrow R$ be defined by
 $f(x) = x^2$. Is f inevitable?

(20 marks)

ii. Let $R_1 = R_2 = R$ the set of all real numbers and $f : R_1 \rightarrow R_2$ be given by the
formula $f(x) = 3x^5 + 3$ and let $g : R_2 \rightarrow R_1$ be given by

$$g(y) = \left(\frac{y}{3} - 1\right)^{1/5}$$

Show that f is a bijection between R_1 and R_2 and g is a bijection between R_2
and R_1 .

(30 marks)

iii. Let the function $h : R \rightarrow R$ be defined by

$$h(x) = \begin{cases} 2x+5 & \text{if } x > 9 \\ x^2 - |x| & \text{if } x \in [-9, 9] \\ x-4 & \text{if } x < -9 \end{cases}$$

Find

a) $h(4)$ b) $h(-6)$ c) $h(h(5))$

(30 marks)

iv. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$ and the function $f: A \rightarrow B$ be defined by

$$f(x) = \frac{x-2}{x-3}$$

Then f is one - one and onto. Find a formula that defines f^{-1}

(20 marks)

06. i. a) Define the product of sets (10 marks)

b) Let $A = \{1, 2, 3\}$, $B = \{2, 4\}$, $C = \{3, 4, 5\}$. Find $A \times B$ and $(A \times B) \times C$
(30 marks)

ii. Let M be a set of integers and \sim be the relation on $M \times M$ defined by $(a_1, a_2) \sim (a_3, a_4)$ if $a_1 + a_4 = a_2 + a_3$. Prove that \sim is an equivalence relation.

(40 marks)

iii. Let R_1, R_2 , and R_3 be the relations on the set \mathbb{N} of positive integers:

R_1 : x is greater than y

R_2 : $x + y = 10$

R_3 : $x + 4y = 10$

Determine which of the relations are

a) Symmetric,

b) Antisymmetric

(20 marks)

SECTION - C

07. i. Show that

a) If $a|b$, then $a||b|$

b) if d and n are positive integer and $d|n$, then $d \leq n$

c) If $p|q$, and $q|r$, then $p|r$.

d) if $3|3x - y^2$, then show that

$$3|3x^2 - 3xy - xy^2 + y^3 + 3x$$

(60 marks)

- ii. Let a and b be integers and $a > 0$. Show that there exists unique integers q and r such that

$$b = qa + r, \quad 0 \leq r < a$$

(40 marks)

08. i. Let integers a and b , not both of which are zero, show that there exist integers x and y such that $\gcd(a, b) = ax + by$

(20 marks)

- ii. Show that

a) for any positive integer m ,
 $\gcd(ma, mb) = m \gcd(a, b)$

b) If $c|ab$ and $(b, c) = 1$, then $c|a$

(30 marks)

- iii. Find $\gcd(125, 962)$ and find integers x_0, y_0 that satisfy the linear equation $125x_0 + 962y_0 = 18$.

Hence give the general solution of the $\gcd(125, 962) = 125x + 962y$ in integers x and y .

(50 marks)

09. i. Let $a, b, c, d, n, m \in \mathbb{Z}$

a) If $a \equiv b \pmod{m}$ and $d|m, d > 0$, then prove that $a \equiv b \pmod{d}$

b) If $a \equiv b \pmod{n_1}$ and $a \equiv c \pmod{n_2}$, then prove that $b \equiv c \pmod{n}$ where the integer $n = \gcd(n_1, n_2)$.

c) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$, then prove that $ac \equiv bd \pmod{m}$

(50 marks)

- ii. Solve the following set of congruence simultaneously (Chinese Remainder Theorem).

$$x \equiv 2 \pmod{5}$$

$$x \equiv 5 \pmod{11}$$

$$x \equiv 7 \pmod{13}$$

$$x \equiv 3 \pmod{17}$$

(50 marks)

- Copyrights reserved -