THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY / BACHELOR OF SOFTWARE ENGINEERING – LEVEL 04
FINAL EXAMINATION – 2011/2012
MPZ4140 / MPZ4160 - DISCRETE MATHEMATICS I

DURATION – ONE AND QUARTER (01-1/4) HOURS

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DATE : 27th February 2012 **TIME: 130 – 1630 hrs.**

Instructions:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Define a statement and which of the following are statements? What are their truth values?

a) "Mahaweli is the longest river in Sri Lanka"
b) "1 = 0"
c) "The number 7"
d) "If I am Buddha, them I am not Buddha."

(20 marks)

ii. Let P be "Pasindu is rich" and let q be "Pasindu is happy" Write each of the following in symbolic form.

a) "Pasindu is poor but happy"
b) "Pasindu is neither rich nor happy"
c) "Pasindu is poor or else he is both rich and unhappy"

(20 marks)

iii. Determine the truth value of each of the following statements.

a) "If 7 < 3, then -3 < -7"
b) "2 + 2 = 4 if and only if 4 +4 = 12"
c) "If 2 + 2 = 4, then 3 + 3 = 8 and 1 + 1 = 2"
d) "If 3 + 3 = 6, then 4 + 4 = 9 if and only if (iff) 1 + 1 = 4"

(20 marks)

iv. Prove De Morgan's Laws by using truth tables.

(20 marks)

v. Let p, q, r be 3 statements. Show that $[p \rightarrow q] \Rightarrow [(p \wedge r) \rightarrow (q \wedge r)]$ is a tautology.

(20 marks)

02. i. Give the converse and contrapositive of the following statements.

a) "If I am Buddha, then I think"

b) "If 2 is an odd number, then all primes are odd"

(20 marks)

ii. Test the validity of the following argument.

"If two sides of a triangle are equal, then the opposite angles are equal. Two sides of a triangle are not equal. Therefore the opposite angles are not equal".

(30 marks)

iii. Determine the truth value and Nagation of each of the following statements (all x ∈ R (real number)).

a) $\forall x, |x| = x$

b) $\forall x,\ x+1 > x$

(20 marks)

iv. Find a counter example for each of the following statements.

a) $\exists\ n \in N,\ 6n+1$ *and* $6n-1$ are not primes.

b) $\exists k \in R,\ \lim_{n \to \infty} \left(1+\frac{1}{n}\right) = k$

(10 marks)

v. Prove that there exists a unique real number x such that $x^3 - 8 = 0$. (Use the direct proof).

(20 marks)

03. i. Show that $p \wedge (p \vee q) \equiv p$, by using laws of the algebra of propositions.

(20 marks)

ii. Using mathematical induction prove that, for each positive integers.

a) $n^3 + 2n$ is divisible by 3.

b) $\sum_{r=1}^{n} r(r+1) = \frac{1}{3} n(n+1)(n+2)$.

(60 marks)

iii. Transcribe the following into logical notation. Let the universe of discourse be the real numbers.

a) For any value of x, x^2 is nonnegative.

b) For every value of x, there is some value of y such that $x \cdot y = 1$.

(20 marks)

SECTION – B

04. i. Let $A = \{x : x \text{ is an odd integer}\}$,

$B = \{x : x \text{ is an integer divisible by} 10\}$

$C = \{x : x \text{ is an integer divisibel by } 5\}$ *and*

$D = \{x : x \text{ is an integer divisible by } 2 \text{ or } 5\}$

Find the elements in each of the following sets.

a) A, B, C, D

b) $A \cap D$

c) $B \cap C$

(30 marks)

ii. Without using venn diagram show that

$A \cup (B \oplus C) = (A \cup B) \oplus (A \cup C)$

$\{Where\ as\ x \oplus y = (x \setminus y) \cup (y \setminus x)\}$

(50 marks)

iii. Let $X = \{1,2,3,4,5\}$, $Y = \{3,4,5,6,7,8\}$ and $Z = \{2,4,6,7,8,9\}$. Find

a) $X \cup (Y \oplus Z)$

b) $X \oplus (Y \cap Z)$

(20 marks)

05. i. Let R be the set of all real numbers and let, $f : R \to R$ be defined by $f(x) = x^2$. Is f inevitable?

(20 marks)

ii. Let $R_1 = R_2 = R$ the set of all real numbers and $f : R_1 \to R_2$ be given by the formula $f(x) = 3x^5 + 3$ and let $g : R_2 \to R_1$ be given by

$g(y) = \left(\frac{y}{3} - 1\right)^{1/5}$.

Show that f is a bijection between R_1 and R_2 and g is a bijection between R_2 and R_1.

(30 marks)

iii. Let the function $h : R \to R$ be defined by

$$h(x)=\begin{cases}2x+5 & if\ x>9\\ x^2-|x| & if\ x\in[-9,9]\\ x-4 & if\ x<-9\end{cases}$$

Find

a) $h(4)$ b) $h(-6)$ c) $h(h(5))$

(30 marks)

iv. Let $A=R-\{3\}$ and $B=R-\{1\}$ and the function $f:A\to B$ be defined by $f(x)=\dfrac{x-2}{x-3}$.

Then f is one – one and onto. Find a formula that defines f^{-1}

(20 marks)

06. i. a) Define the product of sets (10 marks)

b) Let $A=\{1,2,3\},\ B=\{2,4\},\ C=\{3,4,5\}$. Find $A\times B$ and $(A\times B)\times C$

(30 marks)

ii. Let M be a set of integers and ~ be the relation on M x M defined by $(a_1,a_2)\sim(a_3,a_4)$ if $a_1+a_4=a_2+a_3$. Prove that ~ is an equivalence relation.

(40 marks)

iii. Let $R_1, R_2,$ and R_3 be the relations on the set N of positive integers:

R_1: x is greater than y
R_2: $x+y=10$
R_3: $x+4y=10$

Determine which of the relations are

a) Symmetric,
b) Antisymmetric

(20 marks)

SECTION – C

07. i. Show that

a) If $a|b$, then $a|\ |b|$

b) if d and n are positive integer and $d|n$, then $d\le n$

c) If $p|q$, and $q|r$, then $p|r$.

d) if $3|3x-y^2$, then show that

$3|3x^2-3xy-xy^2+y^3+3x$

(60 marks)

ii. Let a and b are integers and $a > 0$. Show that there exists unique integers q and r such that

$$b = qa + r \quad , \quad 0 \leq r < a$$

(40 marks)

08. i. Let integers a and b, not both of which are zero, show that there exist integers x and y such that $gcd\,(a, b) = ax + by$

(20 marks)

ii. Show that

a) for any positive integer m, $gcd(ma, mb) = m \;\; gcd\,(a,b)$

b) If $c|ab$ and $(b,c) = 1$, then $c|a$

(30 marks)

iii. Find gcd (125, 962) and find integers x_0, y_0 that satisfy the linear equation $125\,x_0 + 962\,y_0 = 18$.

Hence give the general solution of the $gcd(125,962) = 125x + 962y$ in integers x and y.

(50 marks)

09. i. Let $a, b, c, d, n, m \in z$

a) If $a \equiv b(\text{mod}\, m)$ *and* $d|m$ $, d > 0$, then prove that $a \equiv b(\text{mod}\ d)$

b) If $a \equiv b(\text{mod}\ n_1)$ *and* $a \equiv c(\text{mod}\ n_2)$, then prove that $b \equiv c(\text{mod}\, n)$ where the integer $n = \gcd(n_1, n_2)$.

c) If $a \equiv b(\text{mod}\ m)$ *and* $c \equiv d(\text{mod}\ m)$, then prove that $ac = bd(\text{mod}\ m)$

(50 marks)

ii. Solve the following set of congruence simultaneously (Chinese Remainder Theorem).

$x \equiv 2(\text{mod}\ 5)$

$x \equiv 5(\text{mod}\ 11)$

$x \equiv 7(\text{mod}\ 13)$

$x \equiv 3(\text{mod}\ 17)$

(50 marks)