19.

THE OPEN UNIVERSITY OF SRI LANKA
B.Sc/B.Ed Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS - LEVEL 04
PSU2182 – EXPERIMENTAL DESIGN
CLOSED BOOK TEST 2006/2007

DURATION: ONE AND HALF-HOURS

DATE: 01 – 03 – 2007	TIME: 3.30pm -5.00pm

ANSWER **ALL** QUESTIONS.

Statistical Tables are provided. Non-programmable calculators are permitted.

1. In a experiment to study the effect of fertilizer on the yield of certain type of paddy, two amounts of fertilizer (5lbs and 10lbs) along with the control (that is no fertilizer) were allocated to 15 plots using completely randomized design so that there were 5 replicates for each treatment. After 3 months total yield (kg) from each plot was measured. The results are as follows. (Here F5 and F10 denote the treatments corresponding to 5lbs and 10lbs of fertilizer respectively.)

	Treatment		
	Control	F5	F10
	18.3	28.1	40.3
	22.6	28.6	35.3
	15.1	31.7	36.5
	11.4	30.3	43.3
	23.4	27.6	37.1
Treatment Total	90.8	146.3	192.5

Grand Total = $\sum x = 429.6$ Total uncorrected sum of squares = $\sum x^2 = 13497.22$

(i) Construct an analysis of variance (ANOVA) table and test whether there is a significant difference between treatment means at 5% significance level.

(ii) Suggest a meaningful independent comparison to compare the effects between
   (a) Fertilizer and Control
   (b) Two amounts of the fertilizer

(iii) Extend the ANOVA table you constructed in part (i), to test the significance of the two comparisons suggest in part (ii). Clearly state your conclusions. Use 5% significance level.

2. An experiment was conducted to compare the effects of four drugs P, Q, R and S (here S is a placebo or inactive substance) on the lymphocyte count in mice. Five litters with each litter containing 4 mice were used and the 4 drugs were randomly allocated to the 4 mice in each litter using a randomized complete block design. Here litters were treated as blocks. The lymphocyte counts measured after 2 hours are as follows.

		Litters					Treatment Totals
		1	2	3	4	5	
Treatment	P	7.1	6.1	6.9	5.6	6.4	32.1
	Q	6.7	5.1	5.9	5.1	5.8	28.6
	R	7.1	5.8	6.2	5.0	6.2	30.3
	S	6.7	5.4	5.7	5.2	5.3	28.3
Block Totals		27.6	22.4	24.7	20.9	23.7	119.3

Total uncorrected sum of squares $= \sum x^2 = 720.51$

(i) Construct an analysis of variance (ANOVA) table and test whether there is a significant difference between treatment means. Clearly state you conclusions. Use 5% significance level.

(ii) Find the Least Significant Difference (LSD) for comparing any two treatment means. (Use 5% significance level)

(iii) Using LSD found in part (ii) above, carryout a pair wise mean comparison to find out which treatment means are significantly different. Clearly explain your answer.

3. Doughnuts absorb fat in various amounts when they are cooked. An experiment is being planned to investigate whether the amount of fat absorbed depends on the type of fat used and the temperature at which Doughnuts were cooked. The experimenter wants to study the effect of three types of fat (F_1, F_2, F_3) and two levels of temperature (T_1, T_2) on the amount of fat absorbed (in grams) by Doughnuts. The resources are available to cook 30 Doughnuts and the experimenter wishes to use completely randomized design with equal replicates.

(i) What is the treatment structure of this experiment?
(ii) How many treatments are there and what are they?
(iii) How many replicates are there in the experiment?
(iv) What is the experimental unit in this experiment?
(v) What is the "response variable" of this experiment?
(vi) Explain how you would use the random number table to do the randomization in this experiment.

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ANSWERS

1.

	Treatment		
	Control	F5	F10
	18.3	28.1	40.3
	22.6	28.6	35.3
	15.1	31.7	36.5
	11.4	30.3	43.3
	23.4	27.6	37.1
Treatment Total	90.8	146.3	192.5

Grand Total $= \sum x = 429.6$ Total uncorrected sum of squares $= \sum x^2 = 13497.22$

(i) Correction Factor (CF) $= \dfrac{(429.6)^2}{15} = 12303.74$

Total Sum of Squares (Total S.S) $= \sum x^2 - \text{CF} = 13497.22 - 12303.74 = 1193.48$

$\text{Trt.S.S} = \dfrac{(90.8)^2 + (146.3)^2 + (192.5)^2}{5} - 12303.74 = 1037.18$

ANOVA				
Source of Variation	d.f	S.S	M.S.S	F-value
Trt. S.S.	2	1037.18	518.59	39.8**
Error S.S	12	156.30	13.03	
Total S.S	14	1193.48		

From the F-table we can get $F_{2,12,5\%} = 3.89$

Therefore it is clear that F-value from the ANOVA table is highly significant and this provides strong evidence that treatment effects are different. Hence we conclude that sufficient evidence is available at 5% significance level to claim that there is significant difference between the effects of the fertilizer on the yield of paddy.

(ii) Let L_1 be the comparison between fertilizer and control. then

$L_1 = 2T_C - T_{F5} - T_{F10}$

Let L_2 be the comparison between two amounts of fertilize. Then

$L_2 = T_{F10} - T_{F5}$

Here T_C, T_{F5} and T_{F10} denote the treatment total for the control, fertilizer (5lbs) and fertilizer (10lbs) respectively.

If we consider the coefficient of the above comparisons we can write

	Control	F5	F10
L_1	2	-1	-1
L_2	0	-1	1

It can be seen that L_1 and L_2 are independent comparisons.

(iii) $L_1 = 2\times(90.8) - 146.3 - 192.5 = -157.2$

Sum of Squares for $L_1 = \dfrac{L_1^2}{r\sum \lambda_i^2} = \dfrac{(-157.2)^2}{5\times 6} = 823.73$

$L_2 = 192.5 - 146.3 = 46.2$

Sum of Squares for $L_2 = \dfrac{L_2^2}{r\sum \lambda_i^2} = \dfrac{(46.2)^2}{5\times 6} = 213.45$

ANOVA				
Source of Variation	d.f	S.S	M.S.S	F-value
Trt. S.S.	2	1037.18	518.59	39.80**
Control vs Fertilizer	1	823.73	823.73	63.22**
Two levels of the fertilizer	1	213.45	213.45	16.38**
Error S.S	12	156.30	13.03	
Total S.S	14	1193.48		

From the F-table we can get $F_{1,12,5\%} = 4.75$

When comparing the F-values from the ANOVA table with the table F-value it is clear that there is a significant difference between the effects of Fertilizer and control as well as between the two amounts of fertilizer on the yield of paddy. Therefore we conclude that

- Application of fertilizer resulted a significant increase in the yield of paddy when compared with the control (no fertilizer). The averages being 33.88kg for fertilizer while 18.16kg for control.

- Application of high amount of fertilizer resulted a significant increase in the yield of paddy. The averages being 38.5kg for 10lbs of fertilizer while 29.26 for 5lbs of fertilizer.

2.

		Litters					Treatment Totals
		1	2	3	4	5	
Treatment	P	7.1	6.1	6.9	5.6	6.4	32.1
	Q	6.7	5.1	5.9	5.1	5.8	28.6
	R	7.1	5.8	6.2	5.0	6.2	30.3
	S	6.7	5.4	5.7	5.2	5.3	28.3
Block Totals		27.6	22.4	24.7	20.9	23.7	119.3

(i) Grand Total $= \sum x = 119.3$ Total uncorrected sum of squares $= \sum x^2 = 720.51$

Correction Factor (CF) $= \dfrac{(119.3)^2}{20} = 711.62$

Total Sum of Squares (Total S.S) $= \sum x^2 - CF = 720.51 - 711.62 = 8.89$

$$\text{Block.S.S} = \frac{(27.6)^2 + (22.4)^2 + (24.7)^2 + (20.9)^2 + (23.7)^2}{4} - 711.62$$

$$= 718.03 - 711.62 = 6.41$$

$$\text{Trt.S.S} = \frac{(32.1)^2 + (28.6)^2 + (30.3)^2 + (28.3)^2}{5} - 711.62$$

$$= 713.47 - 711.62 = 1.85$$

ANOVA				
Source of Variation	d.f	S.S	M.S.S	F-value
Trt. S.S.	3	1.85	0.62	12.4**
Block S.S.	4	6.41	1.60	
Error S.S	12	0.63	0.05	
Total S.S	19	8.89		

Form F-table $F_{3,12,5\%} = 3.49$. Therefore by comparing the F-value from ANOVA table with the table value it is clear that there is a significant difference between the treatment effects. Hence we conclude that there is sufficient evidence at 5% level to claim that effects of the 4 drugs on the lymphocyte counts are different.

(ii) Estimated S.E for comparing any two treatment means $= \sqrt{\dfrac{2s^2}{r}} = \sqrt{\dfrac{2\times(0.05)^2}{5}}$.

$$LSD = \sqrt{\frac{2s^2}{r}} \times t_{12,5\%} = \sqrt{\frac{2\times(0.05)^2}{5}} \times 2.18 = 0.1414 \times 2.18 = 0.31$$

(iii)

Treatment	P	Q	R	S
Mean	6.42	5.72	6.06	5.66

Mean Comparison	Conclusion
$P-Q=6.42-5.72=0.7>LSD(0.31)$	There is a significant difference between the effects of drug P and drug Q on the lymphocyte count.
$P-R=6.42-6.06=0.36>LSD(0.31)$	There is a significant difference between the effects of drug P and drug R on the lymphocyte count.
$P-S=6.42-5.66=0.76>LSD(0.31)$	There is a significant difference between the effects of drug P and drug S on the lymphocyte count.
$R-Q=6.06-5.72=0.34>LSD(0.31)$	There is a significant difference between the effects of drug R and drug Q on the lymphocyte count.
$Q-S=5.72-5.66=0.06<LSD(0.31)$	There is no significant difference between the effects of drug Q and drug S on the lymphocyte count.
$R-S=6.06-5.66=0.4<LSD(0.31)$	There is a significant difference between the effects of drug R and drug S on the lymphocyte count.

3. In this experiment the experimenter wants to test the amount of fat absorbed by doughnuts when cooked using 3 types of fat (F_1, F_2, F_3) at two different temperatures (T_1, T_2).

(i) In this experiment the treatment structure is a two way treatment structure.

(ii) There are 6 treatments. They are $(F_1T_1, F_1T_2, F_2T_1, F_2T_2, F_3T_1, F_3T_2)$

(iii) We have 6 treatments and can cook 30 doughnuts. Therefore using **1** combination of treatment we can cook 5 doughnuts. Hence there are **5 replicates** for each treatment combination.

(iv) Doughnut

(v) Amount of fat absorbed.

(vi) We are going to use CRD for this experiment. First number the doughnuts from 1 – 30. Using random number table read two-digit numbers ignoring numbers greater than 30. Allocate the first treatment combination (F_1T_1) to doughnuts corresponding to first 5 random numbers, 2nd treatment combination (F_1T_2) to doughnuts corresponding to next 5 random numbers and so on.