THE OPEN UNIVERSITY OF SRI LANKA
B.Sc/B.Ed Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS - LEVEL 04
PSU2182 – EXPERIMENTAL DESIGN
OPEN BOOK TEST 2006/2007

DURATION: ONE AND HALF-HOURS

DATE: 05 – 02 – 2007	TIME: 3.30pm -5.00pm

ANSWER **ALL** QUESTIONS.

Statistical Tables are provided. Non-programmable calculators are permitted.

1. A small experiment is being planned to compare the rates of consumption of four different brands of LP gas (say A, B, C and D) used as a car fuel. Suppose there are four cars of particular model and four drivers are available for this experiment. LP gas is supplied for a maximum of four test drives per brand. The overall consumption of fuel may be different from car to car. Also there may be difference in consumption of fuel by each test driver. However it is assumed that consumption of fuel by each driver does not depend on the car he used and also the difference between cars in fuel consumption is same for each driver.

   (i) Suggest a suitable design structure for this experiment. Give reasons for you answer.

   (ii) How many treatments are there and what are they?

   (iii) How many replicates are there in the experiment?

   (iv) What is the "response variable" of this experiment?

   (v) Draw one possible way of allocating treatments to experimental units.

   (vi) Suppose it is given that the fuel consumption is same for each car. Would you still use the design structure mention in part (i) or use a different design structure to carry out this experiment? Justify you answer.

   If you are using a different design structure, briefly explain how you would do it.

2. An experiment need to be design to compare the effect of four drugs P, Q, R and S (here S is a placebo or inactive substance) on the blood sugar level of mice. Suppose the experimenter has 5 litters and each litter contains 4 mice. You are called to assist the experimenter with the designing of this experiment.

(i) Explain what type of design you would use if

(a) all mice are of same age.
(b) within each litter mice are of same age, but between litters mice are different in age.

In each case give reasons for you answer.

(ii) Explain how you use random number table to allocate each treatment to the experimental units for each case given in parts (a) and (b). In each case how many replicates are there?

(iii) What are the advantages and disadvantages of using the design structures you suggested in parts (a) and (b)?

3. A researcher wants to investigate the effect of 4 types of fertilizers (say A, B, C and D) on the growth of lemon trees. Suppose he has 24 lemon plants of equal height and he apply each fertilizer for 6 lemon plants using completely randomized design. After 3 months he measured the increase in height (mm) of each lemon plant. The results are as follows.

Fertilizer			
A	B	C	D
25.12	40.25	18.30	28.05
17.25	35.25	22.60	28.55
26.42	31.98	25.90	33.20
16.08	36.52	15.05	31.68
22.15	43.32	11.42	30.32
15.92	37.10	23.38	27.58

(i) State the hypothesis of interest in this experiment.
(ii) Compute the average increase in height for each fertilizer.
(iii) Construct an analysis of variance (ANOVA) table for this experiment.
(iv) Using the ANOVA table you constructed, test the significance of the hypothesis mentioned in part (i) at 5% significance level. Clearly state you conclusions.

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ANSWERS

1. (i) In this experiment we need to compare the rate of consumption of 4 different LP gases. It is given that rate of consumption differs from car to car as well as from driver to driver. Therefore when designing the experiment we need to control these two sources of variation (cars and drivers). Hence the suitable design structure is a 4×4 Latin Square with cars and drivers representing rows and columns.

   (ii) In this experiment treatments are the 4 different brands of LP gases. Therefore there are 4 treatments.

   (iii) It is given that LP gas is available for maximum of 4 test drives for each brand. Therefore there will be 4 replicates for each treatment.

   (iv) In this experiment one has to measure the rate of consumption of each LP gas. Therefore the response variable is **rate of consumption.**

   (v) In LSD each treatment appear only once in each row and once in each column. Hence one possible way of allocating treatment is

A	B	C	D
B	A	D	C
D	C	A	B
C	D	B	A

   (vi) If fuel consumption is same for each car, we don't need to control the variation that arises due to differences in cars. We need to control only the variation that arises due to differences in drivers. Therefore instead of LSD we can use RCBD with 4 blocks where each block represents a driver.

   Here we can assign each driver 4 test drives using the 4 different brands of LP gases allocated randomly.

2. (i) (a) If all mice are of same age, we can consider the experimental units (mice) as a homogeneous group and we can use CRD to allocate treatments to experimental units.

   (b) If mice are of different age, we have to control the variation that arises due to age difference when designing the experiment. Hence the suitable design structure in this situation is RCBD with 5 blocks where each block represents a litter.

(ii) (a) There are total of 20 mice and we need to allocate 4 treatments (P, Q, R and S) to these mice. To do this first number the mice from 1 to 20. Then using random number table read random numbers of 2 digits ignoring the numbers that are greater than 20. Allocate drug P to mice corresponding to the first 5 numbers, drug Q to mice corresponding to the next 5 numbers and so on until all treatment are allocated to all mice.
There will be 5 replicates for each treatment.

(b) Here each litter will be treated as a block and in each litter there are 4 mice. To allocate treatment to mice, first select a litter. Then number the treatments in a convenient way. For example one can give 1 to P, 2 to Q, 3 to R and 4 to S. Then using random number table read random numbers of 1 digit ignoring numbers greater than 4. Finally assign the treatment (corresponding to each number to each mouse in the selected litter. (For example if the random numbers are 2,4,3,1 then allocate treatment Q to first mouse, S to second mouse, R to third mouse and P to fourth mouse.) Carryout the same process for allocating treatments for the remaining litters.
There will be 5 replicates for each treatment.

(iii)

CRD	
Advantages	Disadvantages
• Flexibility: Any number of treatments or replicates may be used. Also number of replicates may vary from treatment to treatment. All available experimental units can be used. • Statistical Analysis is simple even with unequal replications. • Statistical Analysis remains simple even with missing data.	• Inefficient • Since the randomization is not restricted, the experimental error includes the entire variation. • The experimental units need to be homogeneous.

RCBD	
Advantages	Disadvantages
• More precision than CRD. • No restriction on the number of blocks or the number of treatments. • Statistical analysis is relatively simple even with missing values.	• When the variation among experimental units within a block is large it will produce a larger error term.

3.

	Fertilizer			
	A	B	C	D
	25.12	40.25	18.30	28.05
	17.25	35.25	22.60	28.55
	26.42	31.98	25.90	33.20
	16.08	36.52	15.05	31.68
	22.15	43.32	11.42	30.32
	15.92	37.10	23.38	27.58
Treatment Totals	122.94	224.42	116.65	179.38

(i) The hypothesis of interest

H_0: There is no difference between the treatment effects on the growth of lemon trees.

H_1: There is a difference between the treatment effects on the growth of lemon trees.

(ii) Average increase in height (treatment means)

	Fertilizer			
	A	B	C	D
Treatment Mean	20.49	37.40	19.44	29.90

(iii) For the given data:

Grand Total = 643.39 $\quad \sum x^2 = 18909.05$

$$\text{Correction Factor (CF)} = \frac{(643.39)^2}{24} = 17247.95$$

$$\text{Total Sum of Squares (Total S.S)} = \sum x^2 - \text{CF} = 18909.05 - 17247.95 = 1661.1$$

$$\text{Treatment Sum of Squares (Trt.S.S)} = \frac{\left(\sum x_A\right)^2 + \left(\sum x_B\right)^2 + \left(\sum x_C\right)^2 + \left(\sum x_D\right)^2}{6} - \text{CF}$$

$$\text{Trt.S.S} = \frac{(122.94)^2 + (224.42)^2 + (116.65)^2 + (179.38)^2}{6} - 17247.95$$

$$= 18543.83 - 17247.95$$

$$= 1295.88$$

$$\text{Error Sum of Squares (Error S.S)} = \text{Total S.S} - \text{Trt. S.S}$$

$$= 1661.1 - 1295.88$$

$$= 365.22$$

ANOVA				
Source of Variation	d.f	S.S	M.S.S	F-value
Trt. S.S.	3	1295.88	431.96	23.65**
Error S.S	20	365.22	18.26	
Total S.S	23	1661.10		

From the F-table we can get $F_{3,20,5\%} = 3.10$

Therefore the F-value in ANOVA table is highly significant. Hence the observed difference between treatment effects is significant at 5% level and we conclude that there is a significant difference between the treatment effects on the growth of lemon trees.