THE OPEN UNIVERSITY OF SRI LANKA

DIPLOMA IN TECHNOLOGY– LEVEL 03

FINAL EXAMINATION - 2007

MPZ 3230 – ENGINEERING MATHEMATICS I

DURATION – THREE (03) HOURS

500

DATE : 11th May 2008 **TIME: 9.30 a.m. – 12.30 p.m.**

Answer only six (06) questions selecting at least two from sections A and B.

Instructions:

- **State any assumption you use.**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**

SECTION - A

01. Define the scalar product **a.b** and the vector product **a x b** of two vectors **a** and **b**. The points A,B,C, have coordinates (3,-2,4) (6,3,1) (5,7,3) respectively, referred to rectangular axes Oxyz.

Calculate the products $\overrightarrow{AB}.\overrightarrow{AC}, \overrightarrow{AB}\times\overrightarrow{AC}$ and deduce the values of the cosine of the angle CAB and the area of the triangle ABC.

If the point D has coordinates (2,2,6), find the value $\overrightarrow{AB}\times\overrightarrow{AC}.\overrightarrow{AD}$.

The shortest distance '*l*' between the two lines AC and BD is given by $l=\dfrac{\overrightarrow{AC}\times\overrightarrow{BD}.\overrightarrow{AB}}{\left|\overrightarrow{AC}\times\overrightarrow{BD}\right|}$

Find this **shortest distance** and **comment on** your answer.

02. The position vector, relative to the origin O of an object P, moving in a horizontal plane where $\underline{i}$ and $\underline{j}$ are perpendicular unit vectors, is given by

$$\overrightarrow{OP}=\underline{r}=\frac{Cos\,\omega t}{1+Cos\,\omega t}\underline{i}+\frac{Sin\,\omega t}{1+Cos\,\omega t}\underline{j}$$

Obtain expression for **r** = OP {The magnitude of **r**}.

Prove that $\frac{dr}{dt} = \omega r Tan\frac{\omega t}{2}$

Find the value of OP when $t = \frac{\pi}{2\omega}$

Show that the velocity of the object, $v = \omega r^2(-Sin\omega t\,\underline{i} + (1 + Cos\omega t)\underline{j})$

Hence find the values of t such that the moving direction of the object is perpendicular to the direction of OP.

When $t = \frac{3\pi}{2\omega}$, find the magnitude and direction of the velocity of the object.

03. a) If $A = \begin{bmatrix} Cos\theta + Sin\theta & \sqrt{2}Sin\theta \\ -\sqrt{2}Sin\theta & Cos\theta - Sin\theta \end{bmatrix}$, using the method of mathematical induction, prove that $A^n = \begin{bmatrix} Cos\,n\theta + Sin\,n\theta & \sqrt{2}Sin\,n\theta \\ -\sqrt{2}Sin\,n\theta & Cos\,n\theta - Sin\,n\theta \end{bmatrix}$

, where $n \in Z^+$.

Hence find $\begin{bmatrix} \sqrt{2} & 1 \\ -1 & 0 \end{bmatrix}^4$

b) Without expanding the following determinants at any stage, prove that,

i) $\begin{vmatrix} 1 & bc & bc(b+c) \\ 1 & ca & ca(c+a) \\ 1 & ab & ab(a+b) \end{vmatrix} = 0$

ii) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

04. a) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$

verify that $(AB)^{-1} = B^{-1}A^{-1}$

b) If $f(x) = \begin{bmatrix} Cosx & -Sinx & 0 \\ Sinx & Cosx & 0 \\ 0 & 0 & 1 \end{bmatrix}$

show that $\left[f(x)^{-1}\right] = f(-x)$

c) By using the matrix inversion method, solve

$3x + 2y + 2 = 41$

$2x + y + 2z = 29$

$2x + 2y + 2z = 44$

05. a) On anti-aircraft gun can take maximum of five shots at an enemy place moving away from it. The probabilities of hitting the plane at the first, second, third, fourth and fifth shots are 0.35, 0.30, 0.20, 0.10 and 0.05 respectively. What is the probability that the gun hits the plane?

b) The chance that doctor A will diagnose a disease X correctly is 60%. The chance that a patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70%. A patient of doctor A who had a disease X died. What is the chance that his disease was diagnosed correctly?

06. a) A bag contains six black balls and two white balls. Balls are drawn out one at a time, at random, without replacement, and a white ball is first drawn at the X^{th} draw. Calculate the probabilities $P(X = x)$ for $x = 1,2,3,4,5,6,7$.

Show that the expectation of X is given by $E(X) = 3$ and calculate variation Var (X)

b) A continuous random variable Y has a probability density function defined by,

$$f(y) = \begin{cases} k(5-y) & 0 \le y \le 4 \\ k(y-3) & 4 \le y \le 8 \\ 0 & otherwise \end{cases}$$

Find (i) the value of the constant k

(ii) the expected value Y

(iii) the variance of Y

SECTION – B

07. a) Inverted cone has a depth of 40cm and a base of radius 5cm. Water is poured into the tank at the rate of 1.5π cm^3/min. Find the rate at which level of water in the cone is rising, when a depth is 4cm.

b) The differential equation satisfied by a beam uniformly loaded Wkg/m with one end fixed and the second end subject to tensile force P, is given by $EI\frac{d^2y}{dx^2} = Py - \frac{W}{2}x^2$, where E is the modulus of elasticity, I is the moment of Inertia of the cross section; You can assume $\frac{P}{EI} = n^2$;

(i) Find the complementary function of the given differential equation.

(ii) Find the general solution of the given differential equation.

(iii) Show that the particular solution of the given differential equation (when $\frac{dy}{dx} = 0$ and $y = 0$ at $x = 0$) is

$$y = \frac{W}{Pn^2}(1 - Coshnx) + \frac{Wx^2}{2P}\text{ , where } n^2 = \frac{P}{EI}.$$

08. a) Solve the following ordinary differential equations;

(i) $(x+2)\frac{dy}{dx} = x^2 + 4x - 9$ (ii) $\frac{dy}{dx} = \frac{xy + y}{xy + x}$

(iii) $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$

b) Find the particular solution of the differential equations.

(i) $x(1+y^2) - y(1+x^2)\frac{dy}{dx} = 0$ Given that $y = 0$ when $x = 1$.

(ii) $(1+e^{2x})\frac{dy}{dx} + (1+y^2)e^x = 0$ Given that $y = 1$ when $x = 0$.

09. a) A river is 80 meters wide. The depth y in meters at a distance x meters from one bank is given by the following table.

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find approximately the area of cross section of the river by using

(i) Trapizoidal rule

(ii) Simpson's rule

b) The data given in the following table are about the melting point of an alloy containing Lead and Zinc.

Percentage of Lead in the alloy (%)	50	60	70	80
Melting Point c^0	205	225	248	274

Find the melting point of the alloy containing 54% of Lead by using **Newton's forward interpolation formula**.

10. OPQR is a tetrahedron such that OP = OQ = OR = a and $P\hat{Q}R = \frac{\pi}{2}$.

N is the mid point of PR.

(i) Prove that ON is perpendicular to the plane PQR.

(ii) Find the angle between the planes PQR and OPQ, when the triangle PQR is isosceles and PQ = a.

(iii) Find also the angle between the planes OPQ and OQR.

11. Answer **any two parts** out of four parts (a), (b), (c) and (d).

a) The natural frequency (f) of oscillation of an LRC circuit is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

(i) Find an expression for df

(ii) If C is increased by 1% and L is decreased by 2% , derive an expression for percentage increase or decrease in 'f'.

b) Show that the non linear equation $x^3 - 2x + 0.5 = 0$ has root between $x = 0.1$ and $x = 0.3$.

Use **Netwon – Raphson method**, with $x_1 = 0.3$ as initial value to calculate a more accurate value of the root of the above equation.

Work **correct to 4 decimal places.**

d) Use **Taylor's expansion** to evaluate Sin $60^0\ 30'$, correct to **five decimal places**. Given that $1^0 = 0.01745$ *rad.*

e) Carry out **two steps** of iterations of **Jacobi method** to solve the equations,

$$2x - z = 0$$
$$-x - y + 2z = 0$$
$$-x + 2y = 2$$

Starting with $x^{(0)} = y^{(0)} = z^{(0)} = 0$