THE OPEN UNIVERSITY OF SRI LANKA
B.Sc / B.Ed. Degree Program, Continuing Education Programme
LEVEL 5 –Applied Mathematics
AMU 3181/AME 5181 Fluid Mechanics
Final Examination 2006/2007

Duration: Two and half hours

Date : 20 -04-2007	**Time:10.00 a.m-12.30 p.m**

Answer <u>Four</u> Questions only

01. (a) Define the following terms:

(i) Stream line
(ii) Stream function
(iii) Velocity Potential
(iv) Combination of flows

(b) A Combination of uniform stream $U\underline{i}$ and a doublet of strength $-a^2\,U\underline{i}$ at the origin, flow around a cylinder of a radius a.

(i) Show that the stream function of the flow is given by

$$\psi = U\sin\theta\left(\frac{r^2-a^2}{r}\right). \text{ where } 0\le\theta\le\pi \text{ and } 0<r\le a$$

(ii) Obtain the velocity field from above stream function.
(iii) Find the velocity field on the surface of the cylinder.
(iv) Show that the flow is irrotational.
(v) If there are no body forces, show that the pressure of the cylinder is $P = P_0 + \frac{1}{2}\rho U^2\left(1-4\sin^2\theta\right)$. Here, ρ is the density of the fluid and P_0 is the pressure at a long distance from the cylinder.

02. (a) In the usual notation, show that the pressure gradient $\frac{dP}{dZ}$ due to gravity is given by $\frac{dP}{dZ} = -\rho g$.

(b) The temperature in the Earth's atmosphere, at rest, remains constant and the pressure varies with the density according to Boyle's law $P = K\rho$ where K is a constant. The accleralation due to gravity at a height Z above the ground level is given by $g = \frac{g_0 a^2}{(a+z)^2}$, where a is the radius of earth and g_0 is the value of g at ground level.
Show that the pressure P at a height Z is given by

$$P = P_0 e^{\frac{-g_0 az}{K(a+Z)}}, \text{ where } P_0 \text{ is the value of } P \text{ at ground level.}$$

03. In the usual notation a combination of streaming and swirling flow around a vertically placed cylinder of radius a is given by

$$\psi = Ur\sin\theta - \frac{Ua^2\sin\theta}{r} + \frac{K}{2\pi}\ln\left(\frac{r}{a}\right).$$

Show that, on the surface of the cylinder, the velocity components are given by $U_r = 0$ and $U_\theta = -2U\sin\theta - \dfrac{K}{2\pi a}$.

Show that the pressure distribution P on the cylinder is given by

$$P = C - \frac{1}{2}\rho\left\{4U^2\sin\theta + \frac{2UK\sin\theta}{\pi a} + \frac{K^2}{4\pi^2 a^2}\right\},$$ where C is a constant and ρ is the density of the fluid.

Hence, show that there is no drag force on the cylinder.

04. (a) Assuming the Euler's equation in the form of

$$\frac{\partial U}{\partial t} + \nabla\left(\frac{1}{2}U^2\right) - U\times(\nabla\times U) = F - \frac{1}{\rho}\nabla P,$$ in usual notation,

show that $\dfrac{P}{\rho} + \dfrac{1}{2}U^2 - \Omega =$ constant along the stream lines.

(b) A stream in a horizontal pipe, after passing a contraction in the pipe at which the sectional area is A, is delivered at atmospheric pressure at a place where the sectional area is B. Show that if a side tube is connected with pipe at the former place, water will be sucked up through it into the pipe from the reservoir at a depth,

$$\frac{S^2}{2g}\left(\frac{1}{A^2} - \frac{1}{B^2}\right)$$ below the pipe, where S is the delivery per second

g is the gravity

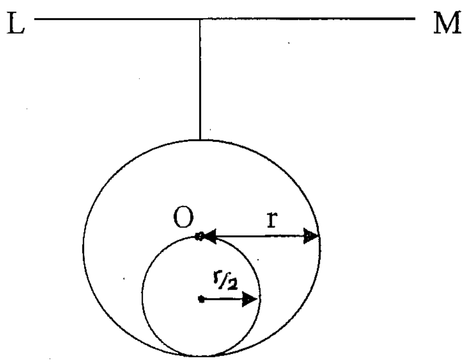
05. (a) Define the following terms

(i) Viscosity (ii) Ideal flow

(iii) Steady flow (iv) Incompressible fluid

(v) Irrotational flow (vi) Unsteady flow

(b) A circular disc of radius $\frac{r}{2}$ is removed from a disc of radius r and the remaining object is vertically submerged in a liquid as illustrated in figure below.



If the water surface level is LM and the centre of pressure of the system is at O, what would be the distance from LM to O?.

06. (a) Show that for a pipe of diameter d and length L, the pressure P loss per unit length due to friction for an incompressible fluid of density ρ, viscosity μ and velocity u is given by

$$\frac{P}{L}=\frac{\rho u^2}{d}\phi(\mathrm{Re}),$$ where $\phi(d,\rho,L,P,\mu,u)=0$ and Re is the Renolds number

(b) Show by the method of dimensional analysis that for the pressure rise ΔP is generated by a pump depends on the impeller diameter D, its rotational speed N, the fluid density ρ and viscosity μ and the rate of discharge $\dot{V}$ may be expressed as

$$\Delta P=\rho N^2D^2\phi\left[\left(\frac{\dot{V}}{ND^3}\right),\left(\frac{\rho ND^2}{\mu}\right)\right],$$ where $\phi(\Delta P,D,N,\rho,\mu,\dot{V})=0$