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The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Final Examination – 2007/2008
Applied Mathematics
AMU 3183/AME 5183 – Numerical Methods II

069

Duration :- Two and Half Hours

Date :- 28-12-2007. **Time :- 1.30 p.m. – 4.00 p.m.**

Answer FOUR Questions Only.

01. (a) Write the Euler–Modified Euler Predictor Corrector formula in the usual notation.

(b) Show that the equation $\frac{dy}{dx} = 3x^2y + 1$, with $y = 0$ when $x = 0$ satisfies Lipschitz condition.

(c) Use the Euler–Modified Euler Method to find y at $x = 0.1$, 0.2 and 0.3 to 4 decimal places.

02. (a) Find coefficient a, b, n, m in order that Runge–Kutta formulae.

$$k_1 = hf(x_0, y_0)$$
$$k_2 = hf(x_0 + mh, y_0 + nk_1)$$
$$y_1 = y_0 + ak_1 + bk_2 \text{ in solving } \frac{dy}{dx} = f(x, y).$$

(b) Determine y at $x = 0.8$ by the second order Runge–Kutta method, given that $\frac{dy}{dx} = \sqrt{x + y}$, $y(0.4) = 0.41$.

Assume the step length $h = 0.2$.

03. (a) Derive normal equations for the least squares straight line fit.

(b) The temperature of a metal was measured at various time intervals during heating and the values are given in the table below.

t (time) min	1	2	3	4
T(temp) °C	70	83	100	124

If the relationship between the temperature and time is of the form $T = be^{t/4} + a$.

(i) Find the least squares estimates for a and b.

(ii) Estimate the temperature (T) of the metal at $t = 6$ min.

04. (a) By applying Newton's forward difference formula,

$p(x_k) = y_0 + k\Delta y_0 + \frac{k(k-1)}{2!}\Delta^2 y_0 + \cdots$ obtain the expression for the first and second derivatives of p_k.

(b) Consider the table of values of $y = \log_e(\sec\theta + \tan\theta)$ given below.

θ	0.0	0.2	0.4	0.6	0.8
y	0.0	0.1679	0.2955	0.4118	0.5460

By considering $\frac{dy}{d\theta}$, construct a table of values for sec θ at $\theta = 0.1, 0.3, 0.5, 0.7$.

Hence find sec θ when $\theta = 0.12$.

(c) Values of a r lynomial function at regular intervals are 20736, 58561, 38416, 50625 65540, 83521, 104976, 130321, 160000. Correct the mistake (if any) in the list.

05. (a) Show that

(i) $\Delta\nabla = \delta^2 = \nabla\Delta = \Delta - \nabla$ where $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$.

(ii) $\Delta E^{-\frac{1}{2}} = \nabla E^{\frac{1}{2}}$.

(iii) $E = e^{hD}$ where $D = \frac{d}{dx}$.

(b) The velocity distribution of a fluid near a flat surface is given below.

x	0.1	0.3	0.5	0.7	0.9
v	0.72	1.81	2.73	3.47	3.98

x is the distance from the surface (cm) and v is the velocity (cm/sec). Obtain th velocity at $x = 0.8$.

06. (a) Write–down with usual notations, Simpson's rule to evaluate $\int_a^b f(x)dx$.

(b) Prove that the truncation error E in using Simpson's rule for the integral $\int_a^b f(x)dx$ i given by $E = -(b-a)\frac{h^4}{180}f^{(iv)}(c)$ where $c \in (a, b)$.

(c) How many subintervals should be taken in the interval (1, 5) in order that the integr $\int_1^5 \log_e x dx$ is calculated accurate to two decimal places using Simpson's rule.