The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 05
Final Examination 2008/2009
Applied Mathematics
AMU 3183/AME 5183 – Numerical Methods II

075

Duration: - Two and Half Hours.

Date: - 23.12.2008 **Time: - 1.00 p.m. – 3.30 p.m.**

Answer Four Questions Only

01. (i) Explain how you would find the Lagrange interpolation polynomial P(x) for the data set $(x_0, y_0), (x_1, y_1), \ldots (x_n, y_n)$.

(ii) With the usual notation, prove that the error of the Lagrange's interpolation $\dfrac{\pi(x) f^{n+1}(c)}{(n+1)!}$, where $c \in (x_0, x_n)$

(iii) The error function $e_r f(x)$ is defined by the integral

$$e_r f(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

Approximate $e_r f(0.08)$ by linear interpolation using the given table of correctly rounded values.

x	0.05	0.10
$e_r f(x)$	0.05637	0.11246

(a) Estimate the total error.

(b) Find the maximal total error for linear interpolation in the interval $0.05 \le x \le 0.10$, using a step size of 0.025.

02.(i) Write down with the usual notation, Simpson's rule to evaluate $\int_a^b f(x)dx$.

(ii) Prove that the truncation error E in using the above rule for the integral $\int_a^b f(x)dx$ is given by $E = \dfrac{-(b-a)h^4 f^{iv}(c)}{180}$; where $c \in (a,b)$ and $h = \dfrac{b-a}{n}$.

(iii) The upward velocity of a rocket can be computed by the following formula.

$v = u\ln\left(\frac{m_0}{m_0 - qt}\right) - gt$; where v is the upward velocity ,u is the velocity at which fuel is expelled relative to the rocket, m_0 is the initial mass of the rocket at time t=0 . q is the fuel consumption rate and g is the downward acceleration of gravity (g=$9.8\ ms^{-2}$).

If u=2200ms^{-1}, $m_0 = 160{,}000kg$ and $q = 2680kgs^{-1}$

(a) Use Simpson's rule to determine how high the rocket will fly in 30s.

(b) Estimate the maximum truncation error.

03. (i) The centered difference operator δ is defined as $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$.Prove the following.

(a) $\Delta = \delta E^{\frac{1}{2}}$ Where Δ is the forward difference operator.

(b) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{\delta^2}{4}}$

(c) $E^{\frac{1}{2}} + E^{-\frac{1}{2}} = \frac{2+\Delta}{\sqrt{1+\Delta}} = \frac{2-\nabla}{\sqrt{1-\nabla}}$;Where ∇ is the backward difference operator.

(ii) An engineer involved in the design of automobiles uses an experimental system for studying the motion of a wide variety of vehicular devices in a full-scale laboratory environment. One particular test involves an accurate measurement of the displacement x of the vehicle as a function of time t. This information is then used to determine the velocity v, the acceleration A, and the rate of change of acceleration F as functions of time.

In a given experiment, the displacement X was measured over a time range of 0s to 10s at step of 0.1s. Some of the results obtained are as follows.

T(s)	0.0	0.1	0.2	0.3	0.4	0.5	0.6
X(s)	0.0	0.8733	1.8224	2.8611	4.0032	5.2625	6.6528

From these data

Compute the v ,A and F at t=0s, employing forward differences with the step size $\Delta(t)$ of 0.1s.

04. (i) Derive normal equations for the least squares straight line fit.

(ii) The presure P of a gass corresponding to various volumes v was recorded as follows.

v(cm3)	50	60	70	90	100
P(kg/cm2)	64.7	51.3	40.5	25.9	7.8

The ideal gas law is given by the equation $PV^r = C$,where r and C are constants.

(a) Find the least squares estimates of r and C from the given data.

(b) Estimate P when $V = 80\text{cm}^3$.

05.(i) Show that the second order Runga–Kutta method, when applied to

$y' = -y, y(0) = 1$ yields $y_m = (1 - h + \frac{h^2}{2})^m$

(ii) Consider the following predictor–corrector pair

$$y_{m+1}^{(0)} = y_m + \frac{h}{2}(y'_m + f(x_m + h, y_m + hy'_m)) \text{ and}$$

$$y_{m+1}^{(i+1)} = y_m + \frac{h}{2}(y'_m + f(x_{m+1}, y'_{m+1})) \quad i = 0,1,2... \quad \text{where } y'_m = f(x_m, y_m)$$

(a) What is the name of this predictor?

(b) What are the Orders of the two formulas?

(c) What advantage does this predictor have over $y_{m+1}^{(0)} = y_{m-1} + 2hf(x_m, y_m)$?

06. (i) Derive the Euler formula for finding the solution of a first order differential equation $\frac{dy}{dx} = f(x,y)$ under the initial condition $y(x_0) = y_0$.

(ii) A metal piece of mass M is released at zero velocity in a liquid and allowed to fall freely under the frictional force. The drag action on the piece due to friction is $M(AV + BV^2)$, where A and B are constants and V is the downward velocity.

(a) Using Euler's method compute the velocity V as a function of time t for $B = 0.1m^{-1}$ and $A = 2s^{-1}$.

(b) Using part(a), find the downward velocity when $t = 0.5s$. Take the down ward acceleration of gravity, g to be 9.8 ms^{-2}.