The Open University of Sri Lanka
Bask/ B.Ed Degree Programme
Final Examination- 2007/2008
Level 5- Applied Mathematics
AMU 3181/AME 5181- Fluid Mechanics

071

Duration : Two and half hours

Date :23.06.2008 **Time : 10.00am-12.30pm**

Answer FOUR Questions only

01. In spherical polar co-ordinate (r, θ), a combination of streaming and swirling flow around a cylinder of radius a is given by $\psi = ur\sin\theta - \dfrac{ua^2\sin\theta}{r} + \dfrac{k}{2\pi}\ln\left(\dfrac{r}{a}\right)$,where the cylinder is vertically placed in the fluid.

(i) Show that, on the surface of the cylinder, the velocity components are given by $u_r = 0$ and $u_\theta = -2u\sin\theta - \dfrac{k}{2\pi a}$.

(ii) Show also that the pressure distribution P on the cylinder is given by $P = C - \dfrac{1}{2}\rho\left\{4u^2\sin^2\theta + \dfrac{2uk\sin\theta}{\pi a} + \dfrac{k^2}{4\pi^2 a^2}\right\}$,where C is a constant and ρ is the density of the fluid.

Hence show that there is no drag force on the cylinder.

02.A steady, two dimensional flow of an inviscid , constant density fluid passes a Cylinder of radius a. Suppose the flow can be modeled by a combination of uniform stream $U\underset{\sim}{i}$ and a doublet of strength $-a^2U\underset{\sim}{i}$.

(i)Show that the stream function is given by $\psi = U\sin\theta\left(\dfrac{r^2 - a^2}{r}\right)$.

(ii)Find the pressure distribution on the surface of the cylinder, given that there are no body forces and the pressure is P_o at a long distance from the cylinder.

(iii) Check whether above combined flow is irrotational or not?

03.(a)(i) In the usual notation, derive the Euler's equation in the form $\frac{\partial \underline{u}}{\partial t}+(\underline{u}.\nabla)\underline{u}=-\frac{1}{\rho}\nabla P+\underline{F}$

(ii)Suppose the flow of a two dimensional inviscid , incompressible fluid is $\underline{u}=2xt\underline{i}+2y\underline{j}$ with body force per unit mass $\underline{F}=2x\underline{i}$. If the pressure is zero at the point (1,0),when $t=3$, find the pressure at the point (2,1).

(b)Find the acceleration at a general point in each of the following two-dimensional velocity fields.

(i) $\underline{u}=U\{(2xt+4x^2)\underline{i}+4y\underline{j}\}$

(ii) $\underline{u}=U(t)y\underline{i}$

(iii) $\underline{u}=U\left(1+\frac{a^2}{r^2}\right)\sin\theta\underline{e}_\theta$,where U and a are constants.

04.(a) Obtain the stream function ψ for a vortex of strength k, where k is a constant.

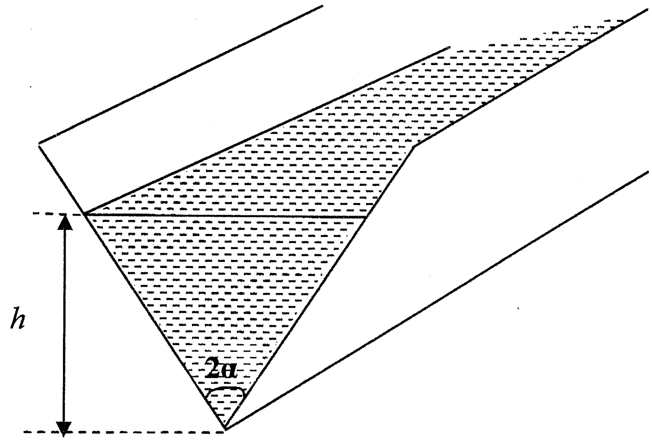
(b) Two lines vortices, each of strength k are situated at $(\pm a,0)$ and a line vortex of strength $-k/2$ is situated at origin.

(i) Show that the fluid motion is stationary and find the general equation of stream lines.

(ii) Show that the stream line that passes through a stagnation point, meets the x-axis at $(\pm b,0)$ when $3\sqrt{3}(b^2-a^2)=4a^3b$.

05.(a) By using the Euler's equation, derive the Bernoulli's equation $\frac{P}{\rho}+\frac{1}{2}u^2-\Omega=\text{constant}$ along a stream line. State the assumptions you make, if any.

(b) The cross section of a horizontal channel is an isosceles triangle with an angle 2α at the apex, as shown in the figure.



(i) write down an expression for the volume flow rate v in terms of velocity u and height h, where h is the height of the triangle.

(ii) By using the Bernoulli's equation, show that the specific energy function is constant.

(iii) For a given constant volume flow rate v, determine a relationship between the critical velocity and critical depth.

06.(a) Show that the torque required to rotate a disc of diameter d at an angular velocity ω in a fluid of density ρ ,viscosity μ is given by, $\tau = d^5\omega^2\rho\,\phi\left(\frac{\rho d^2\omega}{\mu}\right)$.

(b) A disc of diameter 230mm absorbs 160W when rotates in water at a speed of 146 *rad/s*.

(i) What will be the corresponding speed of rotation of a similar disc of diameter 690mm When it rotates under dynamically similar conditions in air?

(ii) Calculate the power absorbed at this speed?

(Take $\mu_w = 101.3\times10^{-5}\,Ns/m^2$, $\rho_a = 1.25\,kg/m^3$, $\mu_a = 1.85\times10^{-5}\,Ns/m^2$)