The Open University of Sri Lanka
B.Sc/B.Ed Degree Programme
Final Examination 2007/2008
Level 03 - Applied Mathematics
AMU 1182/AME 3182 – Conics and Vector Algebra

Duration :- 2 Hours.

Date :- 09.06.2008. **Time:- 01.30 pm. – 03.30 pm.**

Answer FOUR questions only.

01. Consider the line joining the two end points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$.

(i) Find the vector $\overrightarrow{AB}$ and the length of AB.

(ii) Find the direction ratios of $\overrightarrow{AB}$, and hence find the direction cosines of $\overrightarrow{AB}$.

(iii) A, B and C are three points with coordinates (3, –1, 5), (7, 1, 3) and (–5, 9, –1) respectively. L and M are midpoints of AB and BC. Find the length and direction cosines of the line LM.

02. (a) A straight line, is parallel to a vector $\underline{m}$ and passes through a fixed point A, with position vector $\underline{a}$. Find the position vector of any point on the line.

(b) Hence, find the Cartesian equations of the above line.

(c) Find the vector equation of the line through $A(2, -1, 5)$, which intersects perpendicularly, the line whose equations are $\frac{x-3}{1} = \frac{y-1}{2} = \frac{z+1}{2}$.

03. Using vectors find expressions for the

(i) area of a parallelogram,

(ii) area of a Triangle,

(iii) volume of a parallelepiped,

(iv) volume of a tetrahedron.

Hence, find the area of the triangle ABC, whose vertices are at the points $A(1, 2, 1)$, $B(1, 0, 3)$ and $C(-1, 2, -1)$.

04. (a) If $\underline{r}$ is the position vector at any point on a plane, and d is the distance from the origin. Show that the vector equation of the plane is $\underline{r}\cdot\underline{\hat{n}} = d$, where $\underline{\hat{n}}$ is the unit normal to the plane. Hence, find the Cartesian equation of the plane.

(b) Find the vector equation of the plane through the points $A(0, 1, 1)$, $B(2, 1, 0)$ and $C(-2, 0, 3)$

(i) in parametric form,

(ii) in scalar product form.

05. A particle moves so that it's position vector is given by $\underline{r} = \cos\omega t\,\underline{i} + \sin\omega t\,\underline{j}$ where ω is a constant. Show that

(a) The velocity of the particle is perpendicular to $\underline{r}$.

(b) The acceleration $\underline{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin.

(c) $\underline{r} \times \underline{v}$ is a constant vector.

06.(a) Consider the equation $5x^2 + 6xy + 5y^2 + 12x + 4y + 6 = 0$ of a conic. Write down the associated matrix, $\underline{A}$ and find an orthogonal matrix $\underline{P}$ such that $\underline{P}^T\,\underline{A}\,\underline{P} = \underline{D}$, where $\underline{D}$ is a diagonal matrix.

(b) Expressing the equation in part (a) in matrix form and making the transformation $\underline{X} = \underline{P}\,\underline{X}'$ reduced it to $x^2 + 4y^2 + 2\sqrt{2}x + 4\sqrt{2}y + 3 = 0$.

(c) Show further that the equation (b) represents a ellipse.