The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Final Examination – 2007/2008
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

Duration :- Two and a Half Hours

Date :- 23-01-2008. **Time :- 1.00 p.m. –3.30 p.m.**

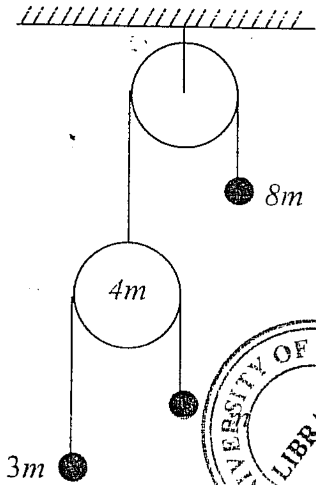
Answer Four Questions Only.

01. Derive velocity and acceleration components in cylindrical polar coordinates.

A particle of mass m moves on the smooth inner surface of the paraboloid of revolution $r^2 = 4az$, whose axis is vertical and vertex downwards. Find the angular momentum of the particle about OZ if it describes a horizontal circle of radius $2a$ with constant speed. While the particle is describing this circle it receives an impulse $m\sqrt{ag}$ along the surface of the paraboloid in a vertical plane through the axis. Show that in the subsequent motion the path of the particle lies between two horizontal planes.

02. In the usual notation, derive Lagrange's equations of motion $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$, $j = 1, 2, \ldots, n$ where T is the kinetic energy of the system.

A light string passes over a fixed smooth pully. It carries a mass of $8m$ at one end, the other end being attached to a smooth pulley of mass $4m$ over which passes a light string whose ends carry masses $3m$ and m.



(*a*) Assuming the system starts from rest and neglecting moments of inertia of the pulleys set up the Lagrangian of the system.

(*b*) Find the accelerations of the moveable pulley and masses.

03. Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

An object is projected vertically downwards with speed v_0. Prove that after time t, the object is deflected east of the vertical by the amount $\frac{1}{3}\omega g \cos\lambda\, t^3 + \omega v_0 \cos\lambda\, t^2$, where λ is the latitude of the point of projection and ω is the angular speed of the earth about its polar axis.

04. (*a*) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(*b*) If a body moves under no forces and if H is the angular momentum about O and T is the kinetic energy of the body then show that H and T are conserved.

(*c*) A rigid body is moving with one of its points O fixed in an inertial frame and the forces acting on the body are equivalent to a single force through O. If the two principal moments of inertia of the body about O are equal. Show that body should rotate with constant angular velocity.

05. (*a*) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(**b**) The Hamiltonian of a dynamical system is given by $H = qp^2 - qp + cp$ where c is a constant. Obtain Hamilton's equations of motion and hence find p and q at time t.

06. (*a*) Define Canonical Transformations.

(*b*) Show that the transformation equations between two sets of equations are $Q = \log\left(1 + \sqrt{q}\cos p\right)$, $P = 2\sqrt{q}\sin p + q\sin 2p$.

(*i*) Show that these transformations are canonical variables if q and p are canonical.

(*ii*) If the generating function $F_3(P, Q)$ given by $P = -\frac{\partial F_3}{\partial Q}$ generates above cannonical transformations, show that $F_3(P, Q) = -(e^Q - 1)^2 \tan p$.