The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Final Examination – 2008/2009
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

078

Duration :- Two and a Half Hours

Date :- 19-01-2009. **Time :- 9.30 a.m. – 12.00 noon.**

Answer Four Questions Only.

01. Show that the acceleration of a particle P moving along a plane curve C is $\ddot{s}\underline{t}+\left(\frac{\dot{s}^2}{\rho}\right)\underline{n}$, where s denotes the arc length along C measured from a fixed point on C, $\underline{t}$, $\underline{n}$ are unit vectors along the tangent and normal at P respectively and ρ is the radius of curvature at P.

A point moves in a plane curve so that its tangential acceleration is constant and the magnitudes of the tangential velocity and normal acceleration are in a constant ratio; find the intrinsic equation of the curve.

02. Show that the velocity and acceleration components of a particle in spherical polar coordinates (r, θ, ϕ) are given by

$$v_r = \dot{r}, \qquad v_\theta = r\dot{\theta}, \qquad v_\phi = r\sin\theta\,\dot{\phi},$$

$$a_r = \ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta, \quad a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) - r\sin\theta\cos\theta\dot{\phi}^2, \quad a_\phi = \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\,\dot{\phi}\right).$$

A particle is attached to one end of a string, of length a, the other end of which is tied to a fixed point O. When the string is inclined at an acute angle α to the downward vertical the particle is projected horizontally and perpendicular to the string with a velocity V. Show that $\dot{\theta}^2 = \left(\frac{V^2}{a^2\sin^2\theta}\right)\left(\cos^2\alpha - \cos^2\theta\right) - \left(\frac{2g}{a}\right)\left(\cos\alpha - \cos\theta\right)$.

03. In the usual notation, derive Lagrange's equations of motion,

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j, \quad j = 1, 2, \ldots, n; \text{ where } T \text{ is the kinetic energy of the system.}$$

A systems of N particles $i = 1, 2, 3, \cdots, N$, with mass m_i, moves around a circle of radius a. The angle between the radius of i^{th} particle and a fixed reference radius is θ_i. The interaction potential for the system is $V = \frac{1}{2}k\sum_{j=1}^{N}\left(\theta_{j+1} - \theta_j\right)^2$, where k is a constant and $\theta_{N+1} = \theta_1 + 2\pi$.

(a) Show that the Lagrangian for the system is $L = \frac{1}{2}a^2\sum_{j=1}^{N} m_j\dot{\theta}_j^2 - V$.

(b) Write down the equation of motion for i^{th} particle and show that the system is in equilibrium when the particles are equally spaced around the circle.

04. Obtain, in a usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

An object of mass m initially at rest is dropped from a height h above the surface of the earth, show that it reaches the earth at a point east of the vertical at a distance $\frac{2}{3}\omega h \cos\lambda\sqrt{2h/g}$, where λ is the nothern latitude of the point of projection and ω is the angular speed of the earth about its polar axis.

05. Derive Euler's equations of motion of a rigid body rotating about a fixed point.

The principal moments of inertia of a body at the centre of mass are A, $3A$, $6A$. The body is so rotated that its angular velocities about the axes are $3n$, $2n$, n respectively. If in the subsequent motion under no force, $\omega_1, \omega_2, \omega_3$ denote the angular velocities about the principal axes at that tine t, show that $\omega_1 = 3\omega_3 = \frac{9n}{\sqrt{5}}\sec u$ and $\omega_2 = 3n\tanh u$ where $u = 3nt + \frac{1}{2}\log_e 5$.

06.(i) Define the Hamiltonian H of a holonomic dynamical system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(ii) The Hamiltonian of a dynamical system is given by $H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, where a, b are constants. Solve the system.