The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2008/2009
AMU 3187/ AME 5187- Mathematical Methods II
Applied Mathematics

Duration: Two and Half Hours.

Date:16.01.2009	**Time: 1.30 p.m.- 4.00p.m.**

Answer FOUR questions only.

(01) (i) If $f(x)$ is a periodic function of x of period p, show that $f(ax)$, $a \neq 0$, is a periodic function of x of period p/a.

(ii) Consider the periodic function f defined by

$$f(x)=\begin{cases} 1, & when \quad 0<x\leq\pi \\ 2, & when \quad \pi<x<2\pi \end{cases}$$

and $f(x+2\pi)=f(x)$.

(a) Sketch the graph of $f(x)$.

(b) Find the Fourier series of $f(x)$.

(c) Using part (b), show that

$$\sum_{r=1}^{\infty}\frac{(-1)^{r+1}}{(2r-1)}=\frac{\pi}{4}.$$

(2) (i) Define each of the following:

(a) odd function,

(b) even function,

(c) half range expansion.

(ii) Find the half range expansions of the function

$$f(x)=\begin{cases} 2x \quad , & 0<x\leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2}<x<1. \end{cases}$$

(iii) Obtain the Fourier series over the interval $-\pi$ to π for the function $h(x) = x(4-x)$.

(3) (i) Define orthogonal functions and orthonormal functions.

(ii) Show that functions 1, $\cos x$, $\sin x$, $\cos 2x$ $\sin 2x$...form an orthogonal set in the interval $-\pi \le x \le \pi$.

(iii) Find the corresponding orthonormal set for the functions of part (ii).

(iv) Verify that the given functions are orthogonal in the given interval with respect to the given weight function $p(x)$ and have the indicated norm.

(a) $g_0(x) = 1, \quad g_1(x) = x, \quad g_2(x) = 2x^2 - 1, \quad -1 \le x \le 1,$

$p(x) = (1-x^2)^{-1/2}$

and $\|g_0\| = \sqrt{\pi}, \|g_1\| = \|g_2\| = \sqrt{\pi/2}$.

(b) $g_0(x) = 1, \quad g_1(x) = 2x, \quad g_2(x) = 4x^2 - 1, \quad -1 \le x \le 1,$

$p(x) = (1-x^2)^{1/2}$

and $\|g_0\| = \|g_1\| = \|g_2\| = \sqrt{\pi/2}$.

(4) For the following boundary value problem:

$$\frac{d^2y}{dx^2} + \mu y = 0$$

$$y(0) = 0 \qquad y(1) = 0,$$

(i) show that it is a Sturm-Liouville problem,

(ii) find the eigenvalues and eigen functions,

(iii) obtain a set of functions which are mutually orthogonal in the interval $0 \le x \le 1$ and

(iv) obtain a corresponding set of orthonormal functions in the interval $0 \le x \le 1$.

(5) The Bessel function of the first kind of order n, $J_n(x)$, is given by the

expansion $J_n(x)=\sum_{r=0}^{\infty}\frac{(-1)^r}{r!\Gamma(n+r+1)}\left(\frac{x}{2}\right)^{n+2r}$, where Γ is the gamma function.

Prove that, for all valid n,

(i) $\frac{d}{dx}\left(x^{-n}J_n(x)\right)=-x^{-n}J_{n+1}(x)$

(ii) $\frac{d}{dx}\left(x^{n}J_n(x)\right)=x^{n}J_{n-1}(x)$.

Hence, show that

$\int xJ_0^2(x)dx=\frac{1}{2}x^2\{J_0^2(x)+J_1^2(x)\}+c$, where c is a constant.

(6) (i) The Rodrigues' formula for the n^{th} degree Legendre polynomial is given by

$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$, -1<x<1.

(a) Using Rodrigues' formula find $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$.

(b) Using part (a) above write down the function $f(x)=2x^3+5x^2+x+1$ in terms of Legendre polynomials.

(ii) Show that the following functions are harmonic:

(a) $u(x,y)=x^2-y^2$,

(b) $u(x,y,z)=3x^2-2y^2+5xy+8xz-z^2$.