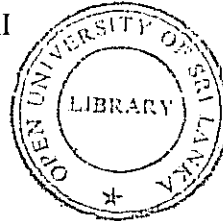


The Open University of Sri Lanka
 B.Sc./B.Ed. Degree Programme
 Final Examination-2008/2009
 AMU 3187/ AME 5187- Mathematical Methods II
 Applied Mathematics



Duration: Two and Half Hours.

Date: 16.01.2009

Time: 1.30 p.m.- 4.00p.m.

Answer FOUR questions only.

(01) (i) If $f(x)$ is a periodic function of x of period p , show that $f(ax)$, $a \neq 0$, is a periodic function of x of period p/a .

(ii) Consider the periodic function f defined by

$$f(x) = \begin{cases} 1, & \text{when } 0 < x \leq \pi \\ 2, & \text{when } \pi < x < 2\pi \end{cases}$$

and $f(x+2\pi) = f(x)$.

(a) Sketch the graph of $f(x)$.

(b) Find the Fourier series of $f(x)$.

(c) Using part (b), show that

$$\sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{(2r-1)} = \frac{\pi}{4}.$$

(2) (i) Define each of the following:

(a) odd function,

(b) even function,

(c) half range expansion.

(ii) Find the half range expansions of the function

$$f(x) = \begin{cases} 2x, & 0 < x \leq \frac{1}{2} \\ 2(1-x), & \frac{1}{2} < x < 1. \end{cases}$$

(iii) Obtain the Fourier series over the interval $-\pi$ to π for the function $h(x) = x(4-x)$.

(3) (i) Define orthogonal functions and orthonormal functions.

(ii) Show that functions $1, \cos x, \sin x, \cos 2x, \sin 2x, \dots$ form an orthogonal set in the interval $-\pi \leq x \leq \pi$.

(iii) Find the corresponding orthonormal set for the functions of part (ii).

(iv) Verify that the given functions are orthogonal in the given interval with respect to the given weight function $p(x)$ and have the indicated norm.

$$(a) \quad g_0(x) = 1, \quad g_1(x) = x, \quad g_2(x) = 2x^2 - 1, \quad -1 \leq x \leq 1,$$

$$p(x) = (1-x^2)^{-1/2}$$

$$\text{and } \|g_0\| = \sqrt{\pi}, \quad \|g_1\| = \|g_2\| = \sqrt{\pi/2}.$$

$$(b) \quad g_0(x) = 1, \quad g_1(x) = 2x, \quad g_2(x) = 4x^2 - 1, \quad -1 \leq x \leq 1,$$

$$p(x) = (1-x^2)^{1/2}$$

$$\text{and } \|g_0\| = \|g_1\| = \|g_2\| = \sqrt{\pi/2}.$$

(4) For the following boundary value problem:

$$\frac{d^2 y}{dx^2} + \mu y = 0$$

$$y(0) = 0 \quad y(1) = 0,$$

(i) show that it is a Sturm-Liouville problem,

(ii) find the eigenvalues and eigen functions,

(iii) obtain a set of functions which are mutually orthogonal in the interval $0 \leq x \leq 1$ and

(iv) obtain a corresponding set of orthonormal functions in the interval $0 \leq x \leq 1$.

(5) The Bessel function of the first kind of order n , $J_n(x)$, is given by the

expansion $J_n(x) = \sum_{r=0}^{\infty} \frac{(-1)^r}{r! \Gamma(n+r+1)} \left(\frac{x}{2}\right)^{n+2r}$, where Γ is the gamma function.

Prove that, for all valid n ,

$$(i) \frac{d}{dx} (x^{-n} J_n(x)) = -x^{-n} J_{n+1}(x)$$

$$(ii) \frac{d}{dx} (x^n J_n(x)) = x^n J_{n-1}(x)$$

Hence, show that

$$\int x J_0^2(x) dx = \frac{1}{2} x^2 \{J_0^2(x) + J_1^2(x)\} + c, \text{ where } c \text{ is a constant.}$$

(6) (i) The Rodrigues' formula for the n^{th} degree Legendre polynomial is given by

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n, \quad -1 < x < 1.$$

(a) Using Rodrigues' formula find $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$.

(b) Using part (a) above write down the function $f(x) = 2x^3 + 5x^2 + x + 1$ in terms of Legendre polynomials.

(ii) Show that the following functions are harmonic:

$$(a) u(x, y) = x^2 - y^2,$$

$$(b) u(x, y, z) = 3x^2 - 2y^2 + 5xy + 8xz - z^2.$$