The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 04
Final Examination – 2008/2009
Applied Mathematics
AMU 2181/AME 4181 – Mathematical Modelling I

074

Duration :- Two and Half Hours.

Date :- 20-01-2009. **Time :- 9.30 a.m. – 12.00 noon.**

Answer Four Questions Only.

01.(a) A factory manufacturers two products X and Y on which the profits earned per unit are Rs. 5 and Rs. 6 respectively. Each product is processed on two machines M_1 and M_2. Product X requires one minute of processing time on M_1 and two minutes on M_2 while Y requires one minute on M_1 and one minute on M_2. Machine M_1 is available for not more than 7 hours and 40 minutes while machine M_2 is available for not more than 10 hours during any working day. Develop a linear programming model and hence use the graphical method to find the optimum number of units of product X and Y to be manufactured to get maximum profit.

(b) Find the maximum value of

$$Z = 50x_1 + 60x_2$$

subject to

$$2x_1 + 3x_2 \le 1500$$
$$3x_1 + 2x_2 \ge 1500$$
$$x_1 \le 400$$
$$x_2 \le 400$$
$$x_1,\ x_2 \ge 0.$$

02. Consider the following linear programming problem.

Minimize $Z = x_1 + 3x_2$

Subject to

$$5x_1 + 4x_2 \ge 20$$
$$3x_1 + 4x_2 \le 24$$
$$x_1 \ge x_2 \ge 0.$$

Identify the feasible region on the (x_1, x_2) space of the problem. Determine the optimal solution from the graph for this problem. What would be the optimal solution if the objective function were to be maximized?

The optimal solution is the corner point of the feasible region which is

(a) nearest to origin for a minimization problem and
(b) farthest from the origin for a maximization problem.

Is the above statement supported by this problem? Is the statement true? Justify your answer.

03. Given the linear programming problem:

Minimize $Z = 8x_1 - 6x_2$

Subject to $x_1 - x_2 \leq 4$

$4x_1 - 2x_2 \leq 8$

$x_1, x_2 \geq 0.$

(a) Solve the problem using simplex algorithm.

(b) Explain the optimum solution if the first constraint of the given problem is replaced by $x_1 - x_2 \geq 4$.

(c) Explain the optimum solution of the given problem if the objective function is replaced by $Z = 8x_1 + 6x_2$.

04. Solve the following problem using big M method:

Minimize $Z = 5x_1 + 3x_2$

Subject to constraints

$2x_1 + 4x_2 \leq 12$

$2x_1 + 2x_2 = 10$

$5x_1 + 2x_2 \geq 10$

$x_1, x_2 \geq 0.$

Verify the solution using the graphical approach.

05. Consider the problem

Minimize $Z = 5x_1 + 8x_2$

Subject to constraints

$x_1 + x_2 \leq 2$

$x_1 - 2x_2 \geq 0$

$-x_1 + 4x_2 \leq 1$

$x_1, x_2 \geq 0.$

What is the dual of the above problem? Solve the primal problem and hence, write the solution of the dual problem.

06. A factory manufacturers three products. Two resources – material and labour – are required to produce these products. Following table gives the requirements of each of the resources for the three products.

Product	Resources (Hours)		Unit Profit (kg)
	Material	Labour	
1	1	5	9
2	1	2	7
3	1	3	5

There are 80 units of material and 250 hours of labour available. In order to determine the optimal product mix which maximizes the total profit, the following linear program was solved.

Maximize $Z = 9x_1 + 7x_2 + 5x_3$

Subject to $x_1 + x_2 + x_3 \leq 80$

$5x_1 + 2x_2 + 3x_3 \leq 250$

$x_1, x_2, x_3 \geq 0$

x_1, x_2 and x_3 are the quantities of product 1, Product 2, product 3 produced. The optimal solution is given in the following table, where x_4, x_5 are slack variables.

Basis	x_1	x_2	x_3	x_4	x_5	values
x_1	1	0	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	30
x_2	0	1	$\frac{2}{3}$	$\frac{5}{3}$	$-\frac{1}{3}$	50
$-Z$	0	0	$-\frac{8}{3}$	$-\frac{20}{6}$	$-\frac{4}{6}$	–620

Using sensitivity analysis, answer the following with respect to the above optimal tableau.

(a) Find the most profitable product mix if the unit profit of product 3 increases to Rs. 8.

(b) What is the range on the profit of product 1 while maintaining the current optimal solution.