The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 03
Open Book Test (OBT) – 2009/2010
Pure Mathematics
PUU 1140 – Logic and Mathematical Proofs

Sample solutions

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1. (i) F
   (ii) T

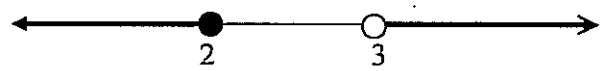
2. (i) PQR is not an isosceles triangle.
   (ii) The number of elements in the set A is strictly less than 5.

3. (i) F
   (ii) T
   (iii) F
   (iv) F

4. Suppose both p, q are true, and r is false. Then $q \vee r$ is true.
   Since both the statements p and $q \vee r$ are true, $p \wedge (q \vee r)$ is true.
   Also since the statement r is false, the statement $(p \vee q) \wedge r$ is false.
   Thus the statement $p \wedge (q \vee r)$ is not logically equivalent to the statement $(p \vee q) \wedge r$.

5. (i) Kamala is not a Buddhist or Inoka is not a Catholic.
   (ii) $x = 4$.

6. Observe that " $x > 2$ implies $x > 3$ " is logically equivalent to " $x \leq 2$ or $x > 3$ ".



7. Suppose the statements, $x = 2$, is true.
   Since $x = 2$, the statement, $x = 2$ or $x = 1$ or $x = -1$, is true.
   This completes the proof.

8. Suppose it is not the case that $x \geq 2$ or $y \geq 2$. Then $x < 2$ and $y < 2$. Hence $x + y < 4$.
   This contradiction completes the proof.
   Name of the method of proof is proof by contradiction.

9. (i) F.
   Let $x = 4$. It is clear that $4 + y^2 \neq 3$ for each $y \in \mathbb{R}$.
   (ii) T.
   Let $n = 1$. Now let $m \in \mathbb{N}$. Then $n^m + m^n = 1^m + m^1 = n + m$.

10. (i) At least one student following PUU1140 has not been to Diyatalawa.
    (ii) Every girl in Bataleeya is not called a Kajukelle.