The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2008/2009
Applied Mathematics
AMU 2185/AME 4185 – Numerical Methods I

Duration: - Two and Half Hours.

Date: - 03.07.2009 **Time: - 10.00 a.m.- 12.30 p.m.**

Answer 4 Questions Only

(1) (a) Briefly explain the following

(i) Absolute error,

(ii) Relative error,

(iii) Truncation error.

(b) The Maclaurin expansion for e^x is given by

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^{n-1}}{(n-1)!} + \frac{x^n}{n!}e^{\varepsilon}, \quad \text{where } 0 < \varepsilon < x$$

Find n such that the series determines $x=1$ correct to eight significant digits.

(c) The criterion for a pass in a certain exam is that the average mark should not be less than 40 marks. Is a candidate who gets an average of 39.5 entitled for a pass? Give reasons for your answer

If the 40 in the criterion is changed to 40.0, what happens to the particular candidate?

(2) (a) Let $f \in [a,b]$ and suppose $f(a)f(b) < 0$. Show that the method of bisection generates a sequence $\{x^{(n)}\}$ approximating the solution, x^* with the property $\left|x^* - x^{(n)}\right| \le \frac{1}{2^n}(b-a),\ n \ge 1$ where n is the number of iterations.

(b) Estimate the number of iterations that will be required to find the solution of $\sqrt{x} = \cos x$ in the interval $[0,1]$ correct to 2 decimal places by the method of bisection.

(c) Find the real root correct to 2 decimal places of the equation $\sqrt{x} = \cos x$ in the interval $[0,1]$ by using the method of Bisection.

(3) (a) (i) What is the geometric interpretation of the Newton's formula for solving $f(x)=0$.

(ii) With the usual notation prove that the condition for convergence of the Newton's method is $|f(x^*)f''(x^*)| < [f'(x^*)]^2$; Where x^* is the solution.

(b) Newton's method for solving the equation $f(x)=c$, where c is a real valued constant is applied to the function.

$$f(x)=\begin{cases}\cos x & when|x|\le 1\\ \cos x+(x^2-1)^2 & when|x|\ge 1\end{cases}$$

For which c is $x_n=(-1)^n$; when $x_0=1$ and the calculation is carried out with no error?

(c) Discuss the advantages and disadvantages of using Newton's method?

(4) (a) Discuss the convergence of the simple iterative method.

(b) The equation $x^2+ax+b=0$ has two real roots α and β show that the iteration method, $x_{k+1}=-\frac{(ax_k+b)}{x_k}$ is convergent, near $x=\alpha$ if $|\alpha|>|\beta|$

(c) The equation $x=f(x)$ is solved by the iteration method, given by $x_{k+1}=f(x_k)$. The solution is required with a maximum error not greater than 0.5×10^{-4}. The computed first and second iterates are given by; $x_1=0.50000$ and $x_2=0.52661$. How many iterations must be performed further if it is known that $|f'(x)|\le 0.53$ for all values of x.

(5) (a) With the usual notation obtain the followings.

(i) $\Delta-\nabla=\Delta\nabla$

(ii) $\Delta+\nabla=\left(\Delta/\nabla\right)-\left(\nabla/\Delta\right)$

(iii) $\frac{1}{2}(E^{1/2}+E^{-1/2})=\frac{2+\Delta}{2\sqrt{1+\Delta}}=\frac{2-\nabla}{2\sqrt{1-\nabla}}$

(b) Complete the following difference table

x	y	**first divided Differences**	**Second divided Differences**	**Third divided Differences**	**Fourth divided Differences**
1.0	0.7651977				
1.3	....	....			
1.6	0.4554022	–0.5489460	–0.1087339		
1.9	....	....	....	....	
2.2	0.1103623	–0.5715210	0.0118183	0.0680685	0.0018251

(c) Using Newton's forward difference formula, find the interpolating polynomial which fits best for the given data.

(6) (a) Let $x_0, x_1, \ldots x_n$ be distinct numbers in the interval $[a, b]$ and $f \in c^{n+1}[a, b]$. Then prove that for each x in $[a, b]$. A number $\varepsilon(x) \in [a, b]$ exists such that

$$f(x) - P_n(x) = \frac{f^{n+1}(\varepsilon(x)}{(n+1)!}\pi(x)$$ where $P_n(x)$ is the Lagrange's interpolation polynomial of degree n and $\pi(x) = (x - x_0)(x - x_1)\ldots(x - x_n)$.

(b) Show that the truncation error of quadratic interpolation in an equidistant table is bounded by $\left(\frac{h^3}{9\sqrt{3}}\right)\max|f'''(\varepsilon)|$;where h is the step size of the equidistance table.

(c) We want to set up an equidistant table of the function $f(x) = x^2 \ln x$ in the interval $5 \le x \le 10$. The function values are rounded to 5 decimals. Find the step size h which is to be used to yield a total error less than 10^{-5} on quadratic interpolation in this table.