The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 04
Final Examination – 2008/2009
Applied Mathematics
AMU 2184/AME 4184 – Newtonian Mechanics

Duration :- Two and Half Hours

Date :- 02-07-2009 **Time :-10.00 a.m. – 12.30 p.m.**

Answer Four Questions Only.

1. A heavy particle of mass m is projected vertically upwards with speed U under gravity in a medium which exerts a resisting force of magnitude mkv^3, where v is the speed of the particle and k is a constant. Let h be the greatest height attained by the particle. Show that

$$h = \frac{V^2}{3g}\left[\ln\left(\frac{\sqrt{U^2 - UV + V^2}}{U+V}\right) + \sqrt{3}\left(\tan^{-1}\frac{2U - V}{\sqrt{3}V} + \frac{\pi}{6}\right)\right], \text{ where } V = \sqrt[3]{\frac{g}{k}}.$$

Find also the speed with which the particle will return to the point of projection.

2. One end of a light elastic string of natural length a and modulus of elasticity $2mg$ is attached to a fixed point O and the other end to a mass m. The particle, initially held at rest at O, is let fall.

(a) Show that the greatest extension of the string during the motion is $a(1+\sqrt{5})/2$.
(b) Find the total time taken by the particle to reach back to the point O again.

3. **(a)** Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being emitted at a rate dm/dt with velocity $\underline{u}$ relative to the particle.

(b) From a rocket which is free to move vertically upwards, matter is ejected downwards with constant relative velocity gT at a constant rate $2M/T$. Initially the rocket is at rest and has mass $2M$, half of which is available for ejection.

(i) Neglecting air resistance and variations in gravitational attraction, show that the greatest upward speed V_{max} is attained when the mass of the rocket is reduced to M and it is given by $V_{max} = (gT/2)(2\log 2 - 1)$.

(ii) Show that the rocket rises to a height $(gT^2/2)(1-\log 2)^2$.

4. (a) With the usual notation show that the velocity and acceleration components in plane polar coordinates are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and

$$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta .$$

(b) A particle P, of mass m, is attached to one end of a light elastic spring of natural length a and modulus $2mg$. The other end of the spring is attached to a fixed point O on a smooth horizontal table on which P is free to move. The particle is projected horizontally with speed $3\sqrt{ag}$ in a direction perpendicular to the spring with the spring at its natural length.

(i) Show that, when the length of the spring is r, in the subsequent motion, its angular speed is $\frac{3}{r^2}\sqrt{ga^3}$.

(ii) Show that the radial speed of the particle is zero when $r = 3a$ and deduce that r satisfies the inequality $a \le r \le 3a$.

5. (a) With the usual notation show that the equation of the central orbit of a particle moving under a central force F per unit mass is given by $\frac{F}{h^2u^2} = \frac{d^2u}{d\theta^2} + u$, where $u = 1/r$.

(b) A particle describes the curve $r^n \cos n\theta = a^n$ under a force F to the pole. Find the law of force

6. A particle is released from rest in a height h above the surface of the earth. Show that it will hit the earth with a speed equal to $\sqrt{\frac{2GMH}{R(R+H)}}$ where R is the radius and M the mass of the earth. Find an expression for time of fall.