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THE OPEN UNIVERSITY OF SRI LANKA
Department Of Civil Engineering
Diploma in Technology (Civil)/ Bachelor of Technology(Civil) – Level 3

CEX3234 – Strength of Materials

FINAL EXAMINATION – 2011/2012

Time Allowed: Three Hours

Date: 2012 - 02 – 27 (Monday) Time: 09.30 – 12.30 hrs

The Paper consists of 8 questions, **Answer** 5 questions only

1. Figure Ql shows a beam of span L that is simply supported at A and B. The beam carries over its entire length a uniformly distributed load of w kN/m (including self weight) and a point load of W at C, which is one third of the span from A.
   a) Sketch the bending moment diagram only for the uniformly distributed load. [4-Marks]
   b) Sketch the bending moment diagram by considering only the point load. [4-Marks]
   c) Now Show that the maximum bending occurs in between C and D when both loads are acting simultaneously. If W takes $2wL$ what will be the value of the maximum moment [12-Marks]

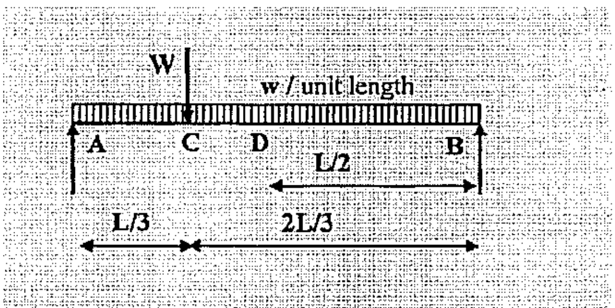

Figure Q1.

2. A cantilever shown in Figure Q2 has a varying cylindrical cross section. At the support the diameter is $2d$ and it becomes d at the free end (L distance away from the support.

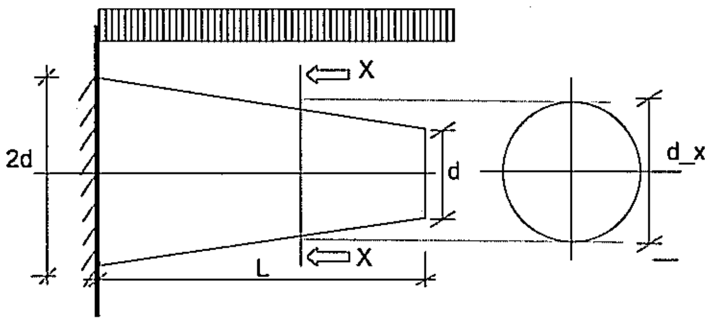

Figure Q2.

   a) Derive the expressions for Bending Moment (BM) and Shear Force (SF) at a distance x away from the free end [6-Marks]
   b) Derive the expression for second moment of area of the section at a x distance away from the free end [2- Marks]

c) Hence determine the location where the bending stress becomes maximum. Structure has the Youngs modulus E and it is straight and vertical when unloaded. [12-Marks]

3 a) Show that shear stress of a beam subjected bending moment M can be derived from the following expression.

$\tau = \frac{V}{Ib}\int_A y dA$ where I is the second moment of area, b is the width of the cross section and y is the distance measured from the neutral axis (Hint: $V=dM/dx$) [8-Marks]

b) Hence show that the shear stress distribution of rectangular section is given by,

$\tau = \frac{V}{2I}(c^2 - y^2)$ where y is the distance measure from the natural axis and $2c$ is the depth of the cross section. [2- Marks].

c) A beam is loaded as shown in the Figure Q3. Determine the shearing stress at a point 3 cm below from the top of the beam at a section 1 m away from the left support. (Hint: You may start by drawing the shear force diagram) [10-Marks]

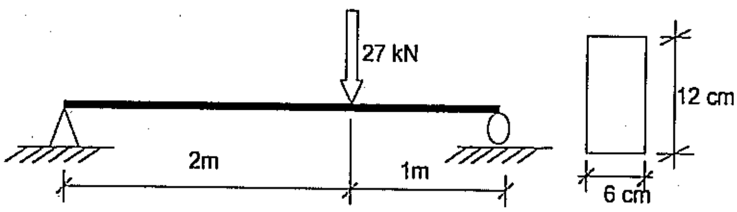

Figure Q3- Loading arrangement

4 a) A steel column is shown in Figure Q4a has a clear height of L and Youngs modulus of E. Calculate the Euler buckling loads

i) When both ends are fixed

ii) When both ends are pinned [4-Marks]

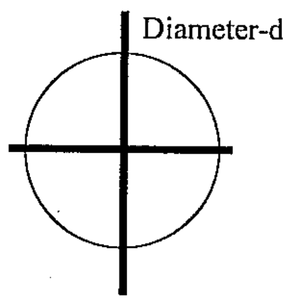

Figure Q4(a)

b) A long thin bar of length L and rigidity EI is pinned at end A, and at end B rotation is resisted by a restoring moment of magnitude λ per radian of rotation at that end. Derive the equation for the axial buckling load P. Neither A nor B can displace in the y-direction,

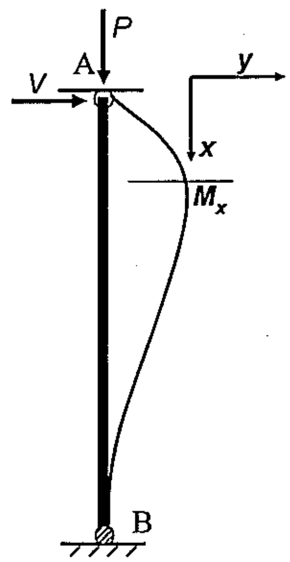

Figure Q4(b)

i) Using the bending moment at x Mx show that the column satisfies the following differential equation

$\frac{d^2y}{dx^2}+\frac{P}{EI}y=\frac{V}{EI}x$ P and V are as shown in the Figure Q4 and EI has the traditional meaning. [4 –Marks]

ii) Show that the general solution for the above differential equation can be given as

$y=C\left[\sin kx-\frac{x}{L}\sin kL\right]$ where $k^2=\frac{P}{EI}$ and C is a constant. [4-Marks]

iii) Show that the restoring moment at B is $M_L=C\lambda\left[k\cos kL-\frac{1}{L}\sin kL\right]$

[4-Marks]

iv) Simplifying, above equations determine the buckling load P [2-Marks]

5. The governing equation for a cylindrical shaft under torsion can be expressed as

$$\frac{T}{J}=\frac{\tau}{r}=\frac{G\theta}{L}$$

a) Explain the meaning of each term. You may use a figure to elaborate your explanations. [6-Marks]

b) Write three assumptions associated with the above torsion formula.[3-Marks]

A steel shaft has the cross section as given in Figure Q5.

c) Determine the angle of twist at the free end [11-Marks]

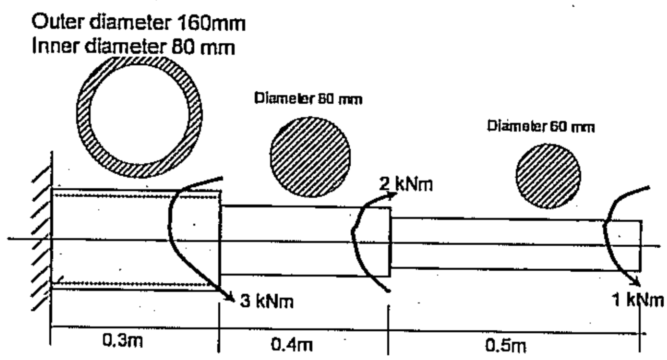

Figure Q5

6. The stress state of an element can be illustrated as in the figure Q 6.

   $\sigma_x = 28$

   $\sigma_y = -14$

   $\sigma_{xy} = 0$

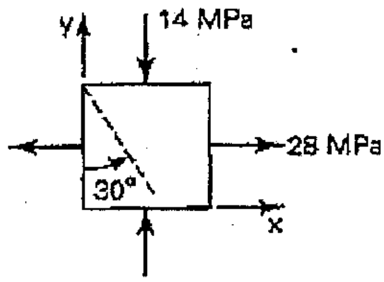

Figure Q6

All stresses are in MPa

a) Draw the Mohr's circle showing the important points. [4-Marks[

b) Determine the major and minor principal stresses and their direction. [4-Marks]

c) Determine the maximum shear stress, corresponding normal stress and the direction. [4-Marks]

d) Determine the normal stress when shear stress is half of the maximum shear stress. [4-Marks]

e) Determine stresses in the plane given above.[4-Marks]

7. Beams constructed with two or more materials having different moduli of elasticity are known as composite materials. To analyze the composite beam the assumption that the cross section remains constant is often applied. Another assumption often used is linear variation of stain $\varepsilon_x = ky$.

   Figure Q7 shows the arrangement of a composite beam.

   a) Using the first principles Show that the beam behaves as having an equivalent bending stiffness of EI which is given in the following expression.

   $EI = E_t I_t + E_s I_s$ where $E_t I_t$ is the bending stiffness of timber and $E_s I_s$ is the bending stiffness of steel respectively. [8 -Marks]

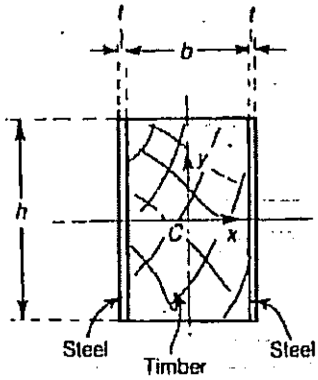

Figure Q7. Composite beam

ii) Hence Prove that the moment carrying capacity of timber can be obtained from the following expression if the applied total moment is M

$$M_t = \frac{M}{1+\dfrac{E_s I_s}{E_t I_t}} \quad \text{[6 –Marks]}$$

iii) Show that the stress at a distance y away from the neutral axis in the steel plate is given by

$$\sigma_s = \frac{My}{I_s + \dfrac{E_t}{E_s} I_t} \quad \text{[4-Marks]}$$

8. A beam fixed at one end and simply supported at the other end carries an uniformly distributed load as shown in Figure Q8
   a) Show that the bending moment at x is equal to $px\text{-}wx^2/2$ where p is the value of support reaction [4-Marks]
   b) Using double integration method the following differential equation can be written
   $$EI\frac{d^2y}{dx^2} = px - w\frac{x^2}{2}$$ where y is the deflection at point **x**
   In order to solve the above differential equation (including p) you need tree boundary conditions. Write three boundary conditions that can be used to solve the above equation [6-Marks]
   c) Show that the support reaction p is equal to $3wL/8$ [7-Marks]
   d) Determine the mid span (x=L/2) deflection and slope [3-Marks]

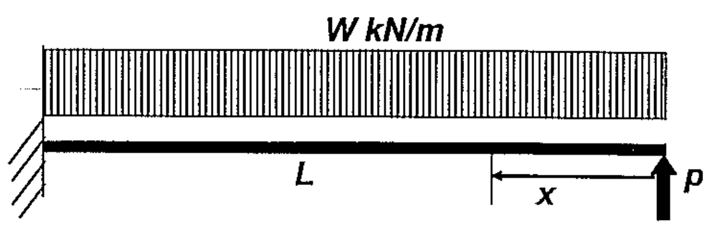

Figure Q8- Loading system