The Open University Of Sri Lanka
B.Sc./B.Ed Degree Programme-Level 05
Final Examination 2009/2010
Applied Mathematics
AMU 3183/AME 5183-Numerical Methods II

Duration:-Two and Half Hours.

Date :- 28.12.2009 Time:-1.00p.m.-3.30p.m.

Answer **Four** Questions Only

01. (i) Explain how you would find the Lagrange interpolation polynomial P(x) for the data set $(x_0, y_0), (x_1, y_1), \ldots (x_n, y_n)$.

(ii) The reduction of diameter ΔD of a trunnion shaft by cooling it through a temperature change of ΔT, for the purpose of shrinking in to a hub is given by $\Delta D = D\alpha\Delta T$, where D is the original diameter ($in.$) and α is the coefficient of thermal expansion at average temperature $(in/in/{}^0F)$.

When the trunnion is cooled from $80\,{}^0F$ to $-108\,{}^0F$, the coefficient of thermal expansion vs. temperature data is given below.

Temperature $T({}^0F)$	Thermal Expansion coefficient, $\alpha(in/in/{}^0F)$
80	6.47×10^{-6}
0	6.00×10^{-6}
-60	5.58×10^{-6}
-160	4.72×10^{-6}
-260	3.58×10^{-6}
-340	2.45×10^{-6}

(a) Determine the value of the coefficient of thermal expansion at $T = -14\,{}^0F$, correct to three decimal places, using third order Lagrange polynomial approximation for the thermal expansion coefficient α.

Find the absolute relative approximate error for the third order polynomial approximation.

(b) The actual reduction in diameter is given by $\Delta D = D\int_{T_r}^{T_f}\alpha dT$, where T_r and T_f are the room temperature(^0F) and temperature of cooling medium(^0F) respectively.
Suppose T_r=80^0F and T_f= -108^0F.

Find the percentage difference in the reduction in the diameter by the above integral formula and the result using the thermal expansion coefficient from part (a).

02. (i) Write down with usual notation, the Simpson's rule to evaluate $\int_a^b f(x)dx$.

(ii) The mass enters or leaves a reactor over a specified time period from t_1 to t_2 is given by

$$M = \int_{t_1}^{t_2} Qcdt \text{, where } Q \text{ is the flow rate.}$$

The outflow chemical concentration from a completely mixed reactor is measured as;

t(min)	0	5	10	15	20	25	30	35	40	45	50	55	60
C (mg/m^3)	10	20	30	40	60	50	80	60	70	60	50	50	60

Assume that the flow rate Q is constant within the first hour.

(a) For an outflow of Q = 10m^3/min, write down the Simpson's rule to estimate the mass of the chemical that exits the reactor within the first hour.

(c) Evaluate the integral using the Simpson's rule for the mass entering the reactor.

03. (i) Define the operators Δ, ∇ and E in the usual notation.

(ii) Prove the following in the usual notation.

(a) $\Delta = E\nabla$

(c) $\nabla(\alpha y_k) = \alpha(\nabla y_k)$

(b) $\Delta(y_k + g_k) = \Delta y_k + \Delta g_k$

(d) $E\Delta = \Delta E$

(iii) Given the set of tabulated points (1,-3), (3,9), (5,30) and (7,132), obtain the value y when x=2 using Newton's divided-Difference formula.

04. (i) Given a set of data $(t_i, h_i), i = 1, 2, \ldots, n,$ derive the formulae for computing the constant coefficients of the function $h = a_0 + a_1 t + a_2 t^2$ by the method of least squares.

(ii) The monthly sales of a company, y (in thousand rupees), over a period of five months starting from June, 2008, where t is taken as 0, are given in the following table:

t	0	1	2	3	4
y	1100	1080	1040	960	840

Estimate the monthly sales in November, 2008 by fitting the function given in part(i) using the method of least squares.

05. (i) Find the coefficients a, b, m and n of the Runge-Kutta formulae of Order 2.

$$K_1 = hf(x_0, y_0),$$
$$K_2 = hf(x_0 + mh, y_0 + nK_1),$$
and $y_1 = y_0 + aK_1 + bK_2$

to approximately solve the initial value problem $\dfrac{dy}{dx} = f(x, y),\ y(x_0) = y_0$.

(ii) Given that y=1 when x=0 and that $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$, use the Runge-kutta fourth order Method with h=0.25 to find the value of y when x=1.

06. (i) Write the Euler-Modified Euler predictor-corrector formula in the usual notation.

(ii) Suppose that $y=y(t)$ is the solution to the initial value problem

$$\frac{dy}{dx} = -\tan(y), \qquad y(0) = 1.$$

Use Euler's method and the trapezium method as a predictor-Corrector pair with one correction at each time step to obtain approximations for $y(0.2)$ and $y(0.4)$. Take the time step to be h =0.2