THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 04
PSU 2182/PSE 4182 – DESIGN AND ANALYSIS OF EXPERIMENTS
CLOSED BOOK TEST 2009/2010

Duration: One and Half Hours.

Date: 02.05.2010	**Time: 4.00 p.m.- 5.30 p.m.**

Answer all questions. Statistical Tables are provided. Non-programmable calculators are permitted.

1. A clinical trial to compare three types of inhaler for treating bronchial asthma involves three groups of five patients. Each group receives one of the inhalers and all patients have their peak expiratory flow rate (PEFR) measured immediately before and one hour after using the inhaler. The improvement in PEFR is shown below:

Type of inhaler		
A	B	C
20	100	100
10	100	80
60	110	120
70	40	90
60	140	80

(i) Construct an analysis of variance (ANOVA) table and test whether the treatments are equally effective. Use 5% level of significance. Clearly state your findings.

(ii) Construct a 95% confidence interval for the difference between treatment means of A and B.

(iii) Using part (ii) or otherwise test whether treatments A and B are equally effective. Use 5% level of significance. Clearly state your findings.

2. The results (coded) of an experiment conducted to investigate the effects of 4 formulations (A, B, C, and D) on the strength of a particular product are given below. Due to practical constraints, the experiment was conducted by three machine operators.

	Formulation			
Operator	A	B	C	D
1	3	4	6	5
2	5	9	8	7
3	7	10	10	9

(i) Write down a model for the response measured on a randomly chosen product. Clearly explain the terms of your model.

(ii) Estimate the difference between the means of formulation A and B.

(iii) Give an estimate for the standard error of the estimate given in part (ii).

3. It is required to study the growth of fish under three types of food (A, B, and C). Food type A is the one which is used currently. B and C are to be introduced if they produce relatively higher growth. Twelve homogeneous tanks and 48 fish (of the same size, age etc.) are available for the experiment. A tank is sufficient for 4 fish. Similar environmental conditions can be provided to all 12 tanks.

(i) Clearly explain how you would design the experiment. (Clearly state the way you allocate treatments and the measurements to be taken)

(ii) Identify the design you suggested in part (i).

(iii) How many linearly independent comparisons could be made in this experiment?

(iv) Write down two meaningful contrasts and test whether they are linearly independent.

Closed Book Test – 2009/2010 – PSU 2182
Answer Guide

1.

	Type of inhaler	
A	B	C
20	100	100
10	100	80
60	110	120
70	40	90
60	140	80
Total 220	490	470

(i). Grand total(G) $=220+490+470$ $=1, 180$

Correlation factor(CF) $=G^2/N$ $=(1180)^2/15$ $=92, 826.67$

Total sum of squire $=\sum_{j=1}^{5}\sum_{i=1}^{3} y_{ij}^2 - CF$ $=(20^2+100^2\cdots)-92, 826.67$ $=18, 373.33$

Treatment SS $=\sum_{i=1}^{3} y_{i.}^2 - CF$ $=\frac{(220^2+490^2+470^2)}{5}-CF$ $=9,053.33$

Error SS = Total SS-Treatment SS $=18, 373.33 -9, 053.33$ $=9, 320$

SOV	SS	Df	MS	F value
Treatment	9, 053.33	2	4526.66	5.82
Error	9, 320.00	12	776.67	
Total	18, 737.33	14		

$F_{tab} = F_{2, 12, 0.05}$ $=3.89$

Since F_{cal} (5.82) > F_{tab} (3.89), The null hypothesis is rejected at 5% level of significance. Thus we can conclude that there is significant different between at least two of the treatments considered.

(ii) $\overline{Y_1} = 220/5 = 44$ $\overline{Y_2} = 490/5 = 98$

95%confidance inerval for the different between treatment means of A and B

$= \overline{Y_B} - \overline{Y_A} \pm t_{\alpha,\theta} S\sqrt{1/r_A + 1/r_B}$ $t_{12,0.05} = 2.18$

$= (98-44) \pm 2.18\sqrt{776.67\left(1/5 + 1/5\right)}$ $=(15.58,92.42)$

(iii) *1st method*

Calculated confidence interval in part (ii) does not contain value zero, so we have enough evidence to assume that the true difference between effectiveness of treatment A and B is non-zero at 5% significance. That means treatment A and B are not equally effective.

2nd method

Least square difference (LSD) $= t_{c\theta}\sqrt{2S^2/n}$ $=2.18\sqrt{2(776.67)/5}$ $=38.42$

Since $\overline{Y_B} - \overline{Y_A}$ > LSD we find that effectiveness of A & B are significantly different at 5% significance level.

2: (i) $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \quad i = 1,2,3;\ j = 1,2,3$

Y_{ij} - Response measured on a product which produce using i^{th} formulation & j^{th} operator.

μ - General mean of response

α_i - Effect of i^{th} formulation

β_j - Effect of j^{th} operator

ε_{ij} - Random error

(ii) Difference between the means of formulation A & B

$$\overline{Y_B} - \overline{Y_A} = \frac{4+9+10}{3} - \frac{3+5+7}{3} = 2.67$$

(iii) Correction factor

$$= G^2/N = (3+5+\ldots+7+9)^2/12 = 574.08$$

Total SS

$$= \sum_{j=1}^{3}\sum_{i=1}^{4} y_{ij} - \text{CF} = (3^2+5^2\ldots) - \text{CF} = 60.92$$

Treatment SS

$$= \sum_{i=1}^{4}\frac{y_{ij}}{3} - \text{CF} = \frac{(15^2+23^2+24^2+21^2)}{3} - \text{CF} = 16.25$$

Block SS

$$= \sum_{j=1}^{3}\frac{y_{ij}}{4} - \text{CF} = 18^2+29^2+36^2/4 - 574 = 41.17$$

Error SS = Total SS-Treatment SS-Block SS =60.92-16.25-41.17 = 3.5

Error mean square (S^2) = 3.5/6 =0.58

Estimate for Standard error

$$= \sqrt{2S^2/r} = \sqrt{2*0.58/3} = 0.62$$

3 (i)

Treatments	replicates
A	4 tanks
B	4 tanks
C	4 tanks

We apply one food type for 4 tanks that means for 16 fish.

Under same condition (the time giving the food is same and quantity also should same). After food, measure the growth (weight of the fish). We can observe the data after a week time (can get different time)

(ii). Completely Random design

(iii). Two linear independent comparison could be made.

(iv). Let T_1 – A Food; T_2 – B Food; T_3 – C Food

i. $T_1 - 1/2(T_2 + T_3)$ -(A and mean of B & C)

ii. $T_2 - T_3$.(B & C)

Whether the independent or not?

{1*0} +{(–1/2)*1} +{(–1/2)*(–1)} =0

Therefore comparisons are independent

(Let $L_1 = \lambda_1 T_1 + \lambda_2 T_2 + \lambda_3 T_3$

and $L_2 = \mu_1 T_1 + \mu_2 T_2 + \mu_3 T_3$

If $\lambda_1\mu_1 + \lambda_2\mu_2 + \lambda_3\mu_3 = 0$ L_1 & L_2 are linearly independent)